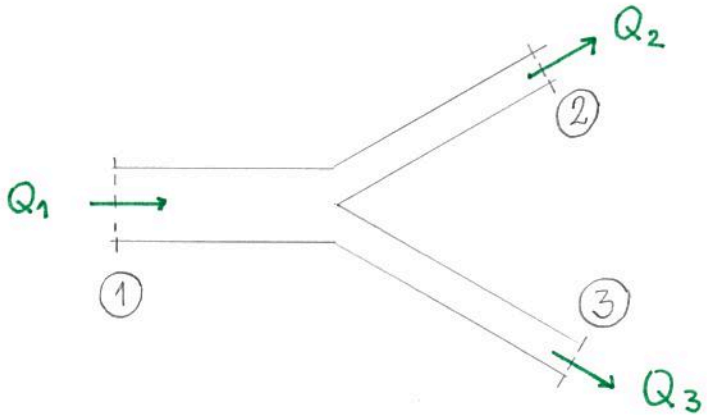
Задатак С1 - петак

Задатак 4.1

(А)

$a = \frac{5+30}{10} = 3{,}5$



Подаци:

$D_1 = (200 + 2 \cdot 30)\,mm = 260\,mm$ $\quad v_1 = 1{,}5\,m/s$

$D_2 = 1{,}2 \cdot 260\,mm = 312\,mm$ $\quad v_2/v_3 = 0{,}75$

$D_3 = 1{,}15 \cdot 312\,mm = 358{,}8\,mm$

ОДРЕДИТИ:

$v_2 = ?$ $\quad v_3 = ?$ $\quad Q_2 = ?$ $\quad Q_3 = ?$

ЈЕДНАЧИНА КОНТИНУИТЕТА

$$Q_1 = Q_2 + Q_3$$

$$(*)\ v_1 A_1 = v_2 A_2 + v_3 A_3$$

ПОВРШИНЕ ПОПРЕЧНИХ ПРЕСЕКА

D[mm]	A [m^2]
260	0,0531
312	0,0764
358,8	0,101

$$A = \frac{D^2 \pi}{4}$$

ПРОТОК У ПРЕСЕКУ 1

$$Q_1 = v_1 \cdot A_1 = 1{,}5\ m/s \cdot 0{,}0531\ m = 0{,}0796\ m^3/s$$

$$Q_1 = 79{,}64\ L/s$$

$$v_2/v_3 = 0{,}75 \Rightarrow v_2 = 0{,}75 \cdot v_3$$

$$(*) \Rightarrow 0{,}0796\ m^3/s = 0{,}75 v_3 \cdot 0{,}0764\ m^2 + v_3 \cdot 0{,}101\ m^2$$

$$v_3 = \frac{0{,}0796\ m^3/s}{0{,}75 \cdot 0{,}0764\ m^2 + 0{,}101\ m^2}$$

$$\boxed{v_3 = 0{,}503\ m/s}$$ БРЗИНА У ПРЕСЕКУ 3

$$v_2/v_3 = 0{,}75 \Rightarrow v_2 = 0{,}75 \cdot 0{,}503\ m/s$$

$$\boxed{v_2 = 0{,}377\ m/s}$$ БРЗИНА У ПРЕСЕКУ 2

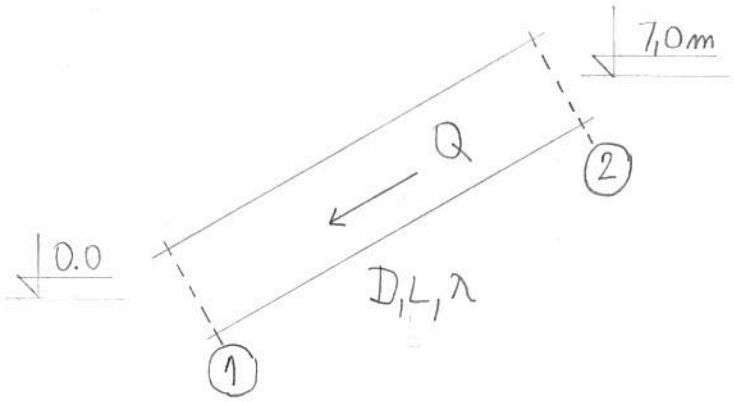
$$Q_3 = v_3 \cdot A_3 = 0{,}503\ m/s \cdot 0{,}101\ m^2 = 0{,}0508\ m^3/s$$

$$\boxed{Q_3 = 50{,}81\ L/s}$$ ПРОТОК У ПРЕСЕКУ 3

$$Q_2 = v_2 \cdot A_2 = 0{,}377\ m/s \cdot 0{,}0764\ m^2 = 0{,}0288\ m^3/s$$

$$\boxed{Q_2 = 28{,}8\ L/s}$$ ПРОТОК У ПРЕСЕКУ 2.

(б)



Подаци:

$L = 5 \cdot 30m = 150m$

$D = (100+30)mm = 130mm$

$\lambda = 0{,}025$

$v = (1 + 5/5)\,m/s = 2\,m/s$

$p_1 = 12 \cdot 30\ kPa = 360\ kPa$

$\rho = 1\,kg/dm^3 = 1000\,kg/m^3$

Одредити:

$\Delta E_{1-2} = ?$ $\quad p_2 = ?$

Енергетска једначина

$$E_2 = E_1 + \Delta E_{1-2}$$

$$\Pi_2 + \frac{v^2}{2g} = \Pi_1 + \frac{v^2}{2g} + \lambda \frac{L}{D} \frac{v^2}{2g}$$

$$\Pi_2 = \Pi_1 + \lambda \frac{L}{D} \frac{v^2}{2g}$$

Губитак енергије између пресека 1 и 2

$$\Delta E_{2-1} = \lambda \frac{L}{D} \frac{v^2}{2g} = 0{,}025 \cdot \frac{150m}{0{,}13m} \cdot \frac{(2\,m/s)^2}{2 \cdot 9{,}81\,m/s^2}$$

$$\boxed{\Delta E_{2-1} = 5{,}881\,m}$$

Да бисмо срачунали притисак у пресеку 2 потребно је да пре тога одредимо пијезометарску коту у пресеку 2

$$\Pi_2 = \Pi_1 + \lambda \frac{L}{D} \frac{v^2}{2g} \quad , \quad \Pi_1 = ?$$

$$p_1 = 360 \times 10^3 Pa \rightarrow \Pi_1 = \frac{p_1}{\rho g} + Z_1 = \frac{360 kPa}{1 kg/dm^3 \cdot 9{,}81 m/s^2} + 0{,}0 m$$

$$\boxed{\Pi_1 = 36{,}697 m}$$

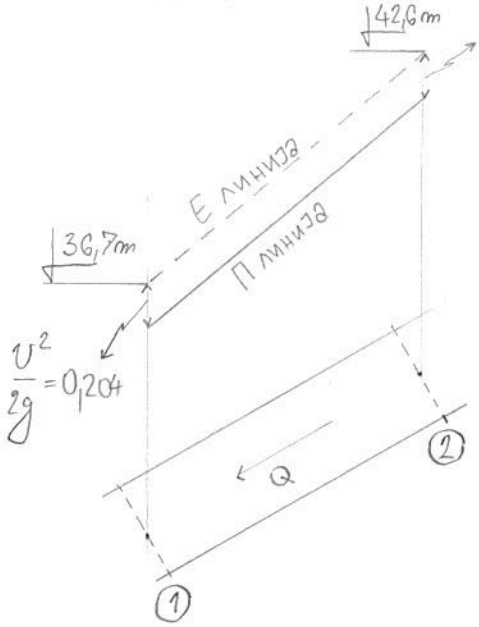
$\Pi_2 = ?$

$$\Pi_2 = 36{,}697 m + 5{,}889 m \qquad \boxed{\Pi_2 = 42{,}578 m}$$

$p_2 = ?$

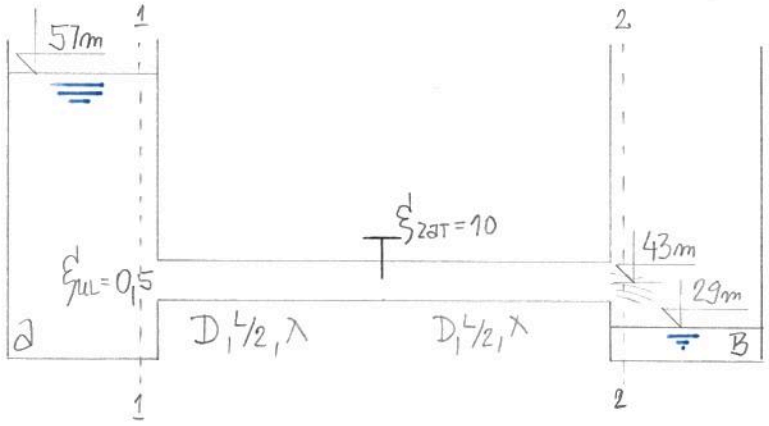
$$p_2 = \rho g (\Pi_2 - Z_2) = 1000 kg/dm^3 \cdot 9{,}81 m/s^2 (42{,}578 - 7) m$$

$$\boxed{p_2 = 349{,}02 kPa}$$



$$\frac{v^2}{2g} = \frac{(2 m/s)^2}{2 \cdot 9{,}81 m/s^2} = 0{,}204 m$$

Задатак 4.2



$a=\frac{30+5}{5}=7$

Подаци:

$L=4\cdot 30m=120m$

$D=(100+30)mm=130mm$

$\lambda=0,022$

$\xi'_{zat}=2\cdot 5=10$

$\Pi_A=(50+7)m=57m$

$\Pi_B=(50-21)m=29m$

$Z_{cevi}=(50-7)m=43m$

Одредити:

Проток кроз цевовод - $Q=?$

Енергетска једначина

$$E_1=E_2+\Delta E_{1-2}$$

$$\Pi_A+\frac{v_A^2}{2g}=Z_c+\frac{v_c^2}{2g}+\xi'_{ul}\cdot\frac{v_c^2}{2g}+\lambda\frac{L/2}{D_c}\frac{v_c^2}{2g}+\xi'_{zat}\cdot\frac{v_c^2}{2g}+$$

$\frac{v_A^2}{2g}\to 0$ — брзине течења у резервоару су занемариве

$$+\lambda\frac{L/2}{D_c}\cdot\frac{v_c^2}{2g}$$

$$\Pi_A=Z_c+\frac{v_c^2}{2g}\left(1+\xi'_{ul}+\lambda\frac{L/2}{D_c}+\xi'_{zat}+\lambda\frac{L/2}{D_c}\right)$$

$$v_c^2 = \frac{Q^2}{A_c^2}, \quad A_c^2 = \left(\frac{D_c^2 \pi}{4}\right)^2 = (0{,}0133\,m^2)^2 = 1{,}762 \times 10^{-4}\,m^4$$

$$\Rightarrow \Pi_a = z_c + \frac{Q^2}{2g A_c^2}\left(1 + \xi'_{ul} + \lambda \frac{L}{D_c} + \xi'_{zat}\right)$$

$$\Rightarrow Q = \sqrt{\frac{(\Pi_a - z_c)\, 2g A_c^2}{1 + \xi'_{ul} + \lambda \frac{L}{D_c} + \xi'_{zat}}}$$

$$Q = \sqrt{\frac{(57-43)m \cdot 2 \cdot 9{,}81\,m/s^2 \cdot 1{,}762 \times 10^{-4}\,m^4}{1 + 0{,}5 + 0{,}022 \frac{120\,m}{0{,}13\,m} + 10}}$$

$$Q = 0{,}039\,m^3/s$$

$$\boxed{Q = 39{,}0\,l/s} \quad \Rightarrow v_c = \frac{0{,}039\,m^3/s}{0{,}0133\,m^2}$$

$$v_c = 2{,}933\,m/s$$

$$\frac{v_c^2}{2g} = \frac{(2{,}933\,m/s)^2}{2 \cdot 9{,}81\,m/s^2} = 0{,}438\,m$$

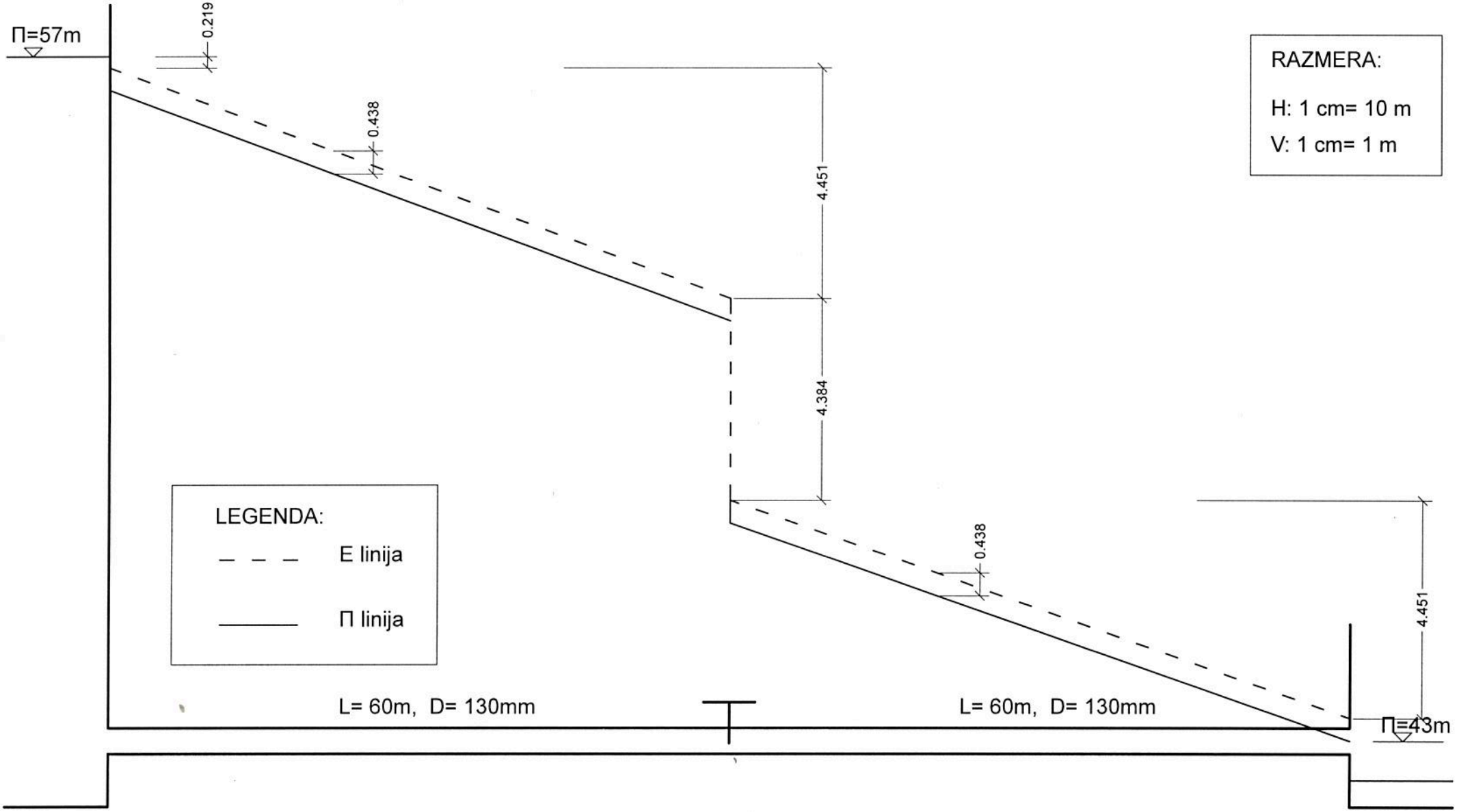
Енергетска једначина

$$57m = 43m + 0{,}438m + 0{,}5 \cdot 0{,}438m + 0{,}022 \frac{60m}{0{,}13m} \cdot 0{,}438m + 10 \cdot 0{,}438m + 0{,}022 \cdot \frac{60m}{0{,}13m} \cdot 0{,}438m$$

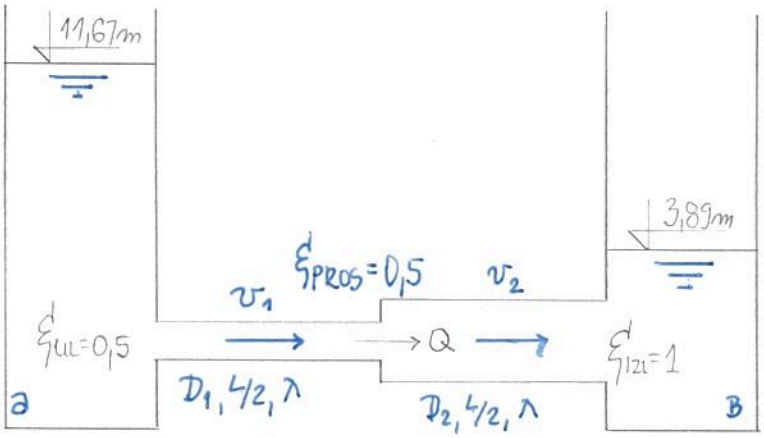
$$57m = 43m + 0{,}438m + 0{,}219m + 4{,}451m + 4{,}384m + 4{,}451m$$

- 0,438 m — брзинска висина
- 0,219 m — губитак на улазу у цев
- 4,451 m — линијски губитак до затварача
- 4,384 m — губитак на затварачу
- 4,451 m — линијски губитак од затварача до излаза из цеви

PIJEZOMETARSKA I ENERGETSKA LINIJA zadatak 4.2



Задатак 4.3



$a=\frac{5+30}{3}=11,6$

Подаци:

$\Pi_a = 11,67m$

$L = 4 \cdot 30m = 120m$

$D_1 = \left(100 - \frac{30}{3}\right)mm = 90mm$

$D_2 = \left(100 + \frac{30}{3}\right)mm = 110mm$

$\lambda = 0,035$

$\xi_{PROS} = 0,25 + \frac{5}{20} = 0,5$

Потребно одредити

проток кроз цевовод

Q

Енергетска једначина

$$E_a = E_B + \Delta E_{a-B}$$

$$\Pi_a + \frac{v_a^2}{2g} = \Pi_B + \frac{v_B^2}{2g} + \xi_{ul}\frac{v_1^2}{2g} + \lambda\frac{l/2}{D_1}\frac{v_1^2}{2g} + \xi_{PROS}\frac{v_2^2}{2g} + \lambda\frac{l/2}{D_2}\frac{v_2^2}{2g} + \xi_{IZL}\frac{v_2^2}{2g}$$

($v_a = 0$, $v_B = 0$)

$$\Pi_a = \Pi_B + \left(\xi_{ul} + \lambda\frac{l/2}{D_1}\right)\frac{v_1^2}{2g} + \left(\xi_{PROS} + \lambda\frac{l/2}{D_2} + \xi_{IZL}\right)\frac{v_2^2}{2g}$$

$$v_1 = \frac{Q}{A_1}, \quad A_1 = \frac{0,09^2\pi}{4} = 6,36 \times 10^{-3} m^2$$

$$v_2 = \frac{Q}{A_2}, \quad A_2 = \frac{0,11^2\pi}{4} = 9,503 \times 10^{-3} m^2$$

$$\Pi_2 = \Pi_B + \frac{Q^2}{2g}\left(\frac{\xi_{uL} + \lambda\frac{L/2}{D_1}}{A_1^2} + \frac{\xi_{PROS} + \lambda\frac{L/2}{D_2} + \xi_{IZL}}{A_2^2}\right)$$

$$Q = \sqrt{\frac{(\Pi_2 - \Pi_B)\cdot 2g}{\frac{\xi_{uL} + \lambda\frac{L/2}{D_1}}{A_1^2} + \frac{\xi_{PROS} + \lambda\frac{L/2}{D_2} + \xi_{IZL}}{A_2^2}}}$$

$$Q = \sqrt{\frac{(11{,}67m - 3{,}89m)\cdot 2\cdot 9{,}81\,m/s^2}{\frac{0{,}5 + 0{,}035\frac{60m}{0{,}09m}}{(6{,}362\cdot 10^{-3})^2 m^2} + \frac{0{,}5 + 0{,}035\frac{60m}{0{,}11m} + 1}{(9{,}503\cdot 10^{-3})^2 m^2}}}$$

$$Q = 0{,}0137\,m^3/s$$

$$\boxed{Q = 13{,}67\,L/s}$$

$\rightarrow v_1 = 2{,}148\,m/s \rightarrow \frac{v_1^2}{2g} = 0{,}235m$

$\rightarrow v_2 = 1{,}438\,m/s \rightarrow \frac{v_2^2}{2g} = 0{,}105m$

ГУБИЦИ ЕНЕРГИЈЕ ПО ЈЕДИНИЦИ ТЕЖИНЕ ДУЖ d-B

$$\xi_{uL}\cdot\frac{v_1^2}{2g} = 0{,}5\cdot 0{,}235 = 0{,}1175m$$

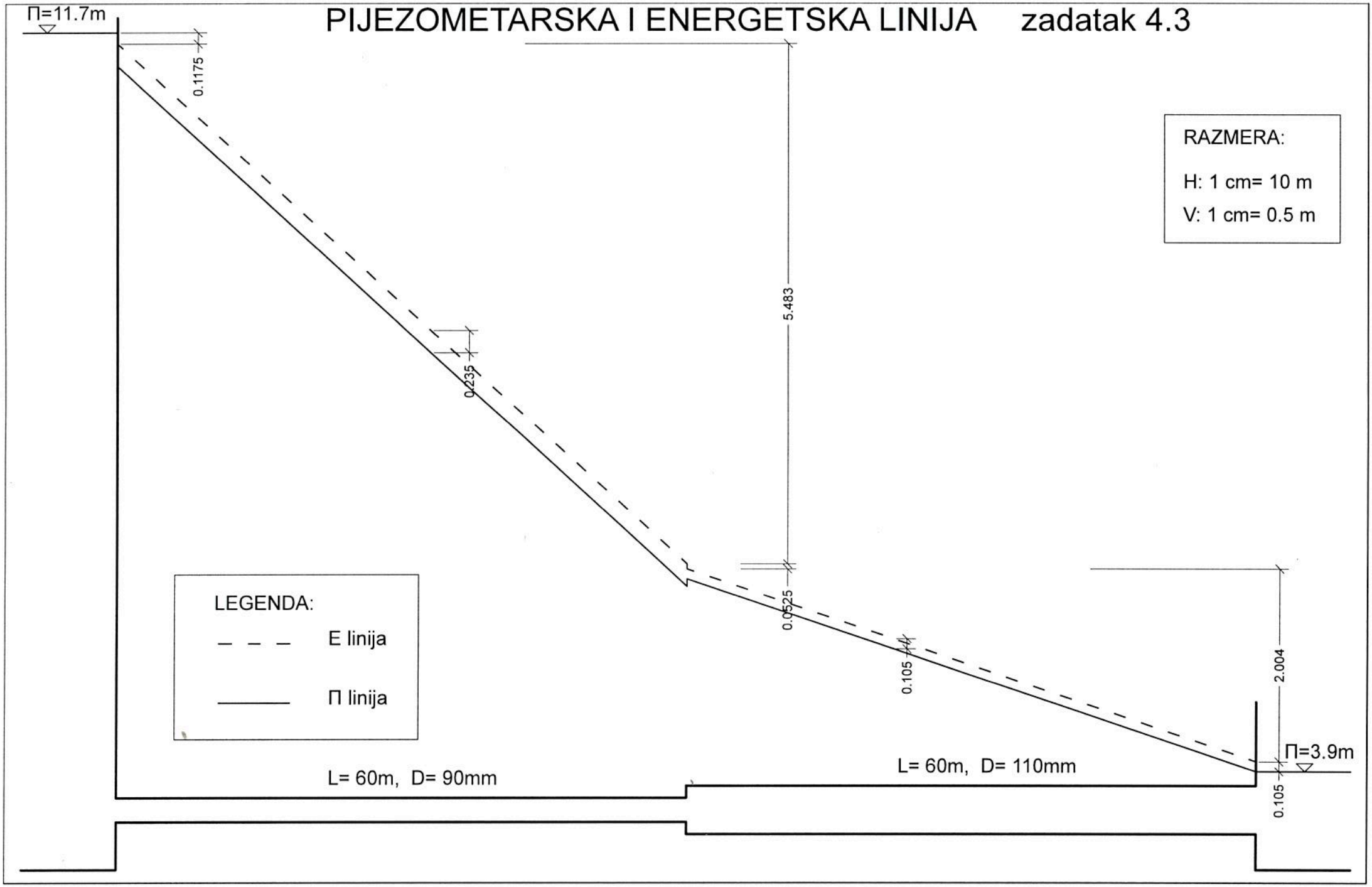
$$\lambda\frac{L/2}{D_1}\cdot\frac{v_1^2}{2g} = 0{,}035\cdot\frac{60m}{0{,}09m}\cdot 0{,}235m = 5{,}483m$$

$$\xi_{PROS}\cdot\frac{v_2^2}{2g} = 0{,}5\cdot 0{,}105m = 0{,}0525m$$

v_2 → НИЗВОДНА БРЗИНА!

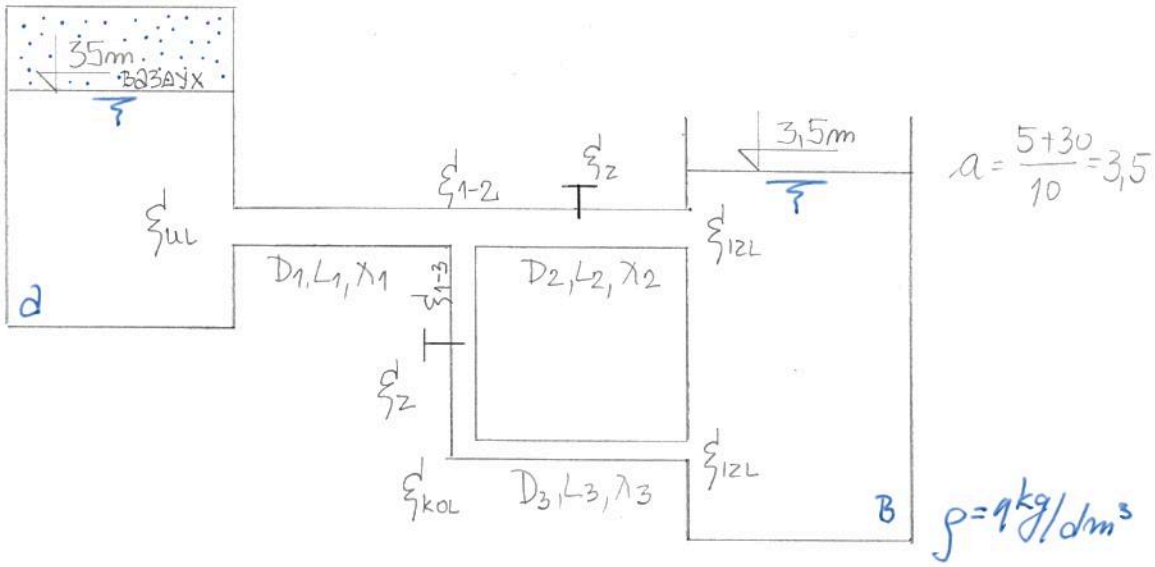
$$\lambda\frac{L/2}{D_2}\cdot\frac{v_2^2}{2g} = 0{,}035\cdot\frac{60m}{0{,}11m}\cdot 0{,}105m = 2{,}004m$$

$$\xi_{IZL}\cdot\frac{v_2^2}{2g} = 1\cdot 0{,}105m = 0{,}105m$$


PIJEZOMETARSKA I ENERGETSKA LINIJA zadatak 4.3
Π=11.7m
0.1175
0.235
5.483
0.0525
0.105
2.004
Π=3.9m
0.105
RAZMERA:
H: 1 cm= 10 m
V: 1 cm= 0.5 m
LEGENDA:
E linija
Π linija
L= 60m, D= 90mm
L= 60m, D= 110mm

Задатак 4.4

$\Pi_a = 59{,}46\,m$



$a = \frac{5+30}{10} = 3{,}5$

$\rho = 1\,kg/dm^3$

Подаци:

$\lambda_1 = 0{,}02$	$\xi_z = 1{,}5$	$D_1 = 300\,mm$	$L_1 = 2 \cdot 30\,m = 60\,m$
$\lambda_2 = 0{,}035$	$\xi_{KOL} = 0{,}2$	$D_2 = 250\,mm$	$L_2 = 2 \cdot 30\,m = 60\,m$
$\lambda_3 = 0{,}035$	$\xi_{1-2} = 0{,}25$	$D_3 = 200\,mm$	$L_3 = 3 \cdot 30\,m = 90\,m$
	$\xi_{1-3} = 0{,}35$		

$\Pi_B = 3{,}5\,m$ $\quad p_{vaz} = 8 \cdot 30\,kPa = 240\,kPa = 240 \times 10^3\,Pa$ $\quad Z_a = 35\,m$

Прорачун Π_a

$$\frac{p_a}{\rho g} = \Pi_a - Z_a \rightarrow \Pi_a = \frac{p_a}{\rho g} + Z_a = \frac{240 \times 10^3\,Pa}{10^3\,kg/m^3 \cdot 9{,}81\,m/s^2} + 35\,m$$

$$\boxed{\Pi_a = 59{,}46\,m}$$

Једначина континуитета за рачву

(1) $Q_1 = Q_2 + Q_3, \quad v_1 A_1 = v_2 A_2 + v_3 A_3$

Бернулијева једначина

(1-2) $E_a = E_B + \Delta E_{a-B}$

(2) $\Pi_a + \frac{v_a^2}{2g} = \Pi_B + \frac{v_B^2}{2g} + \xi_{ul} \cdot \frac{v_1^2}{2g} + \lambda_1 \frac{L_1}{D_1} \frac{v_1^2}{2g} + \xi_{1-2} \frac{v_2^2}{2g} + \lambda_2 \frac{L_2}{D_2} \frac{v_2^2}{2g} + \xi_{2ST} \cdot \frac{v_2^2}{2g} + \xi_{12L} \cdot \frac{v_2^2}{2g}$

(1-3) $E_a = E_B + \Delta E_{a-B}$

(3) $\Pi_a + \frac{v_a^2}{2g} = \Pi_B + \frac{v_B^2}{2g} + \xi_{ul} \cdot \frac{v_1^2}{2g} + \lambda_1 \frac{L_1}{D_1} \frac{v_1^2}{2g} + \xi_{1-3} \frac{v_3^2}{2g} + \lambda_3 \frac{L_3}{D_3} \cdot \frac{v_3^2}{2g} + \xi_{2ST} \frac{v_3^2}{2g} + \xi_{12L} \frac{v_3^2}{2g}$

$$59,46 - 3,5 = \left(0,5 + 0,02 \frac{60}{0,3}\right) \frac{Q_1^2}{2gA_1^2} + \left(0,25 + 0,035 \frac{60}{0,25} + 1,5 + 1\right) \cdot \frac{Q_2^2}{2gA_2^2}$$

$$55,96 = 45,9038\, Q_1^2 + 235,8501\, Q_2^2$$

$$59,46 - 3,5 = \left(0,5 + 0,02 \frac{60}{0,3}\right) \frac{Q_1^2}{2gA_1^2} + \left(0,35 + 0,035 \frac{90}{0,2} + 1,5 + 0,2 + 1\right) \cdot \frac{Q_3^2}{2gA_3^2}$$

$$55,96 = 45,9038\, Q_1^2 + 970,8656\, Q_3^2$$

(2) – (3)

$$\Rightarrow 0 = 235{,}8501\, Q_2^2 - 970{,}8656\, Q_3^2$$

$$\Rightarrow Q_2^2 = 4{,}11645\, Q_3^2$$

Из (2) и (1) $\Rightarrow$ $55{,}96 = 45{,}9038\,(Q_2+Q_3)^2 + 235{,}8501\, Q_2^2$

$$\Rightarrow \boxed{\begin{aligned} Q_2 &= 406{,}8\ l/s \\ Q_3 &= 200{,}5\ l/s \\ Q_1 &= 607{,}3\ l/s \end{aligned}} \quad \Rightarrow \frac{v_2^2}{2g} = 3{,}5m \quad \Rightarrow \frac{v_1^2}{2g} = 3{,}756m$$

ГУБИЦИ ЕНЕРГИЈЕ ДУЖ 1-2

$$\xi_{ul} \cdot \frac{v_1^2}{2g} = 1{,}878m$$

$$\lambda_1 \frac{L_1}{D_1} \cdot \frac{v_1^2}{2g} = 15{,}024m$$

$$\xi_{1-2} \cdot \frac{v_2^2}{2g} = 0{,}875m$$

$$\lambda_2 \frac{L_2/2}{D_2} \cdot \frac{v_2^2}{2g} = 14{,}7m$$

$$\xi_2 \cdot \frac{v_2^2}{2g} = 5{,}25m$$

$$\lambda_2 \frac{L_2/2}{D_2} \cdot \frac{v_2^2}{2g} = 14{,}7m$$

$$\xi_{12L} \cdot \frac{v_2^2}{2g} = 3{,}5m$$

PIJEZOMETARSKA I ENERGETSKA LINIJA zadatak 4.4