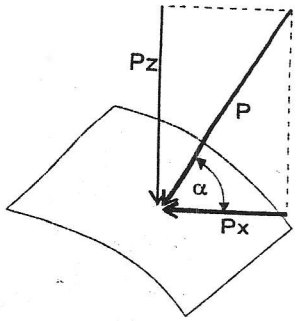
Vežba H2 – ODREĐIVANJE HIDROSTATIČKE SILE NA POVRŠ

Ukupna sila **P** na površ A se razlaže na:

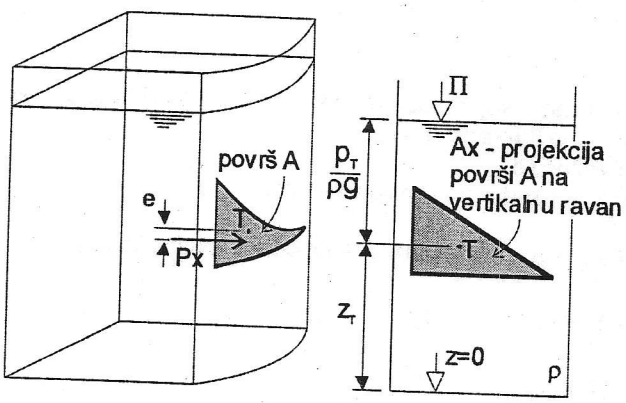
– vertikalnu komponentu **Pz** i

– horizontalnu komponentu **Px**.



HORIZONTALNA KOMPONENTA SILE Px

1. Slučaj kada je projekcija površi na vertikalnu ravan (Ax) proizvoljnog oblika



– **Intenzitet sile:** $Px=p_T Ax$

p_T - pritisak u težištu projekcije Ax: $p_T = \rho g (\Pi - z_T)$

– **Mesto delovanja:** pošto se pritisak menja po visini, mesto delovanja sile je

* ispod težišta (e<0), ukoliko je $p_T>0$

* iznad težišta (e>0), ukoliko je $p_T<0$

$$e=\frac{-\rho g \cdot Iyy}{Px}$$

Iyy – centrifugalni momenat inercije u odnosu na horizontalnu osu koja prolazi kroz težište (vidi prilog u elaboratu)

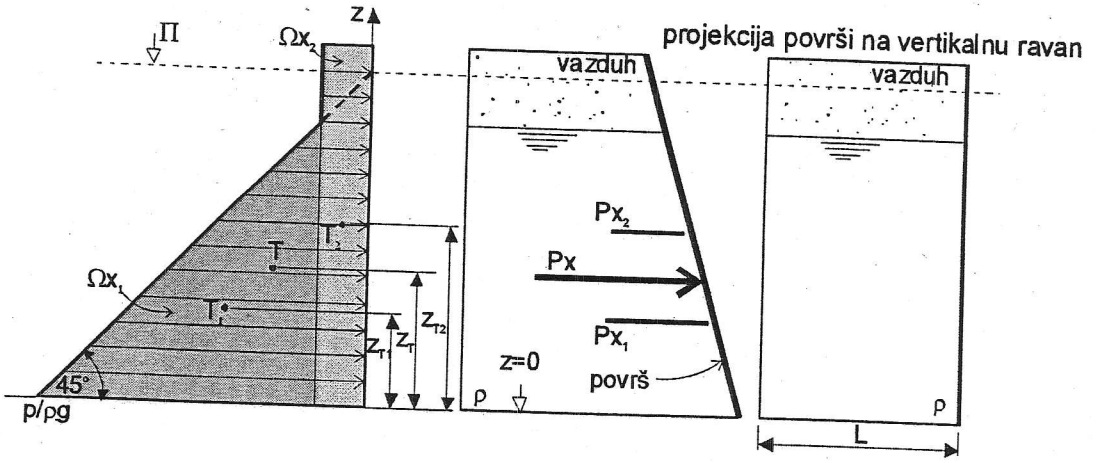
– **Smer delovanja:**

* ka površi, ukoliko je $p_T>0$

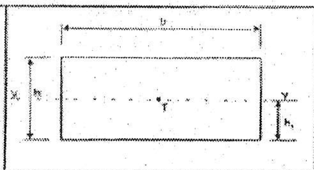
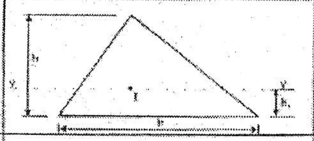
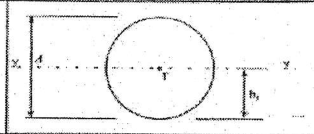
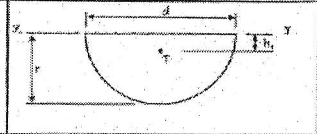
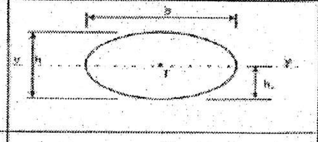
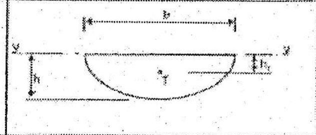
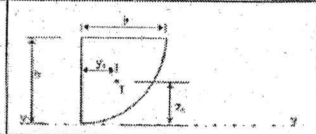
* od površi, ukoliko je $p_T<0$.

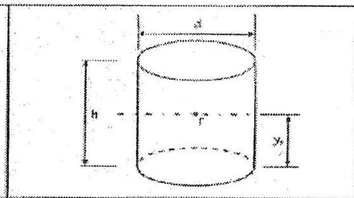
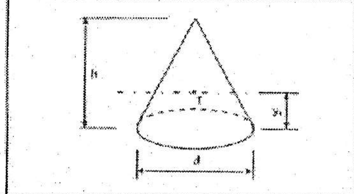
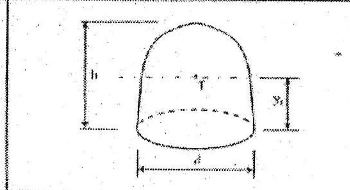
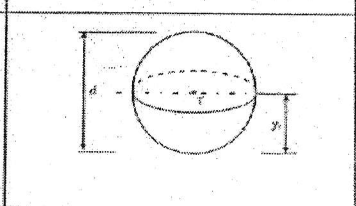
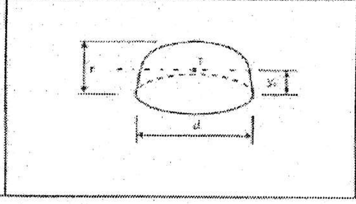
2. Slučaj kada je projekcija površi na vertikalnu ravan Ax pravougaonog oblika – ravanski zadatak

- **Prvi način određivanja horizontalne komponente hidrostatičke sile:**



- nacrta se dijagram visine pritiska (objašnjenje u vežbi H1). Površina dijagrama je Ωx
- površina Ωx se izdeli na delove (Ωx_1, Ωx_2...)
- izračunaju se horizontalne komponente sile po obrascu $Px_i=\rho_i g L \Omega x_i$ (u primeru: $Px_1=\rho g L \Omega x_1$ $Px_2=\rho g L \Omega x_2$)
- mesto delovanja komponenti sile je na kotama težišta površina Ωx_i
- smer delovanja se određuje na isti način kao i kod neravanskih zadataka
- ukupna horizontalna sila dobija se vektorskim sabiranjem komponenti: $\overrightarrow{Px} = \sum \overrightarrow{Px_i}$
- mesto delovanja određuje se preko jednakog momenta oko proizvoljne tačke: $z_{Px} = \frac{\sum Px_i z_{Ti}}{Px}$

		površina	položaj težišta	momenat inercije
PRAVOUGAONIK		$b\,h$	$h_t=\frac{h}{2}$	$I_x=\frac{b\,h^3}{12}$
TROUGAO		$\frac{b\,h}{2}$	$h_t=\frac{h}{3}$	$I_x=\frac{b\,h^3}{36}$
KRUG		$\frac{\pi d^2}{4}$	$h_t=\frac{d}{2}$	$I_x=\frac{\pi d^4}{64}$
POLUKRUG		$\frac{\pi d^2}{8}$	$h_t=\frac{4r}{3\pi}$	$I_x=\frac{\pi d^4}{128}$
ELIPSA		$\frac{\pi\,b\,h}{4}$	$h_t=\frac{h}{2}$	$I_x=\frac{\pi\,b\,h^3}{64}$
POLUELIPSA		$\frac{\pi\,b\,h}{4}$	$h_t=\frac{4h}{3\pi}$	$\frac{\pi b^3 h}{64}$
PARABOLA		$\frac{2}{3}\,b\,h$	$z_t=\frac{3h}{5}$ $y_t=\frac{3b}{8}$	$I_x=\frac{2\,b\,h^3}{7}$

		zapremina	položaj težišta
VALJAK		$\frac{\pi d^2 h}{4}$	$y_t=\frac{h}{2}$
KUPA		$\frac{1}{3}\,\frac{\pi d^2 h}{4}$	$y_t=\frac{h}{4}$
PARABOLOID		$\frac{1}{2}\,\frac{\pi d^2 h}{4}$	$y_t=\frac{h}{3}$
SFERA		$\frac{\pi d^3}{6}$	$y_t=\frac{d}{2}$
POLUSFERA		$\frac{\pi d^3}{12}$	$y_t=\frac{3r}{8}$

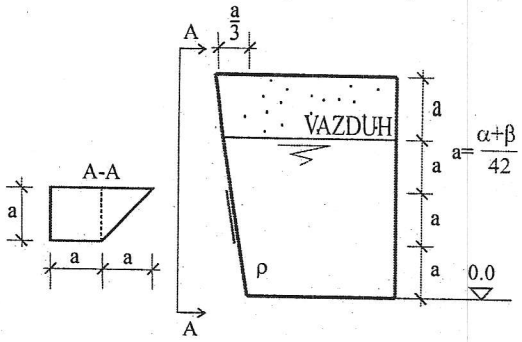
JEDNOSTAVNI ZADACI

ZADATAK 2.1 (*Napomena: slike kotirati u metrima, a ne u opštim brojevima*)

U rezervoaru prikazanom na skici nalazi se tečnost gustine $\rho=1+\frac{\alpha}{4}$ kg/dm^3 i vazduh pod pritiskom.

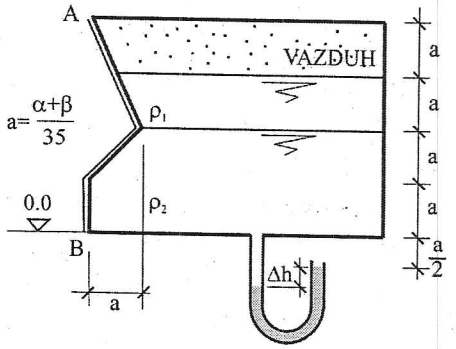
Pijezometarska kota tečnosti iznosi $\Pi=3.2a$ m. Ukoliko se gustina vazduha zanemari, izračunati:

1. horizontalnu komponentu hidrostatičke sile (intenzitet, smer i mesto delovanja) na poklopac složenog oblika koji je prikazan na slici (ucrtati silu na skici), i
2. pritisak u vazduhu iznad tečnosti.



ZADATAK 2.2

U zatvorenom rezervoaru prikazanom na slici nalaze se dve tečnosti gustina $\rho_2=1$ kg/dm^3 i $\rho_1=0.75a$ kg/dm^3 kao i vazduh pod pritiskom. Gustina vazduha se zanemaruje. Ukoliko je pijezometarska kota fluida ρ_1, $\Pi_1=2.7a$ m, nacrtati dijagram visine pritisaka i izračunati horizontalnu komponentu hidrostatičke sile na zid A-B (intenzitet i mesto delovanja) ravanskim postupkom (ucrtati silu na skici). Širina rezervoara je 1m. Takođe, odrediti čitanje Δh na živinom manometru ($\rho_ž=13.6$ kg/dm^3).



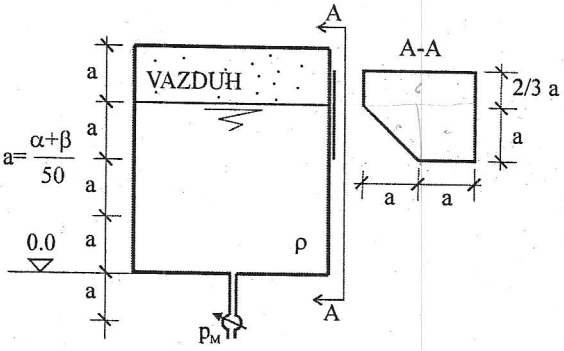
SLOŽENI ZADACI

ZADATAK 2.3

U rezervoaru prikazanom na skici nalazi se tečnost gustine $\rho=1.1$ kg/dm^3 i vazduh pod pritiskom. Manometar prikazan na slici pokazuje pritisak od $p_M=+\frac{\beta}{5}$ kPa. Izračunati:

1. pritisak u vazduhu iznad tečnosti,
2. horizontalnu komponentu hidrostatičke sile (intenzitet, smer i mesto delovanja) na poklopac složenog oblika koji je prikazan na slici (ucrtati silu na skici).

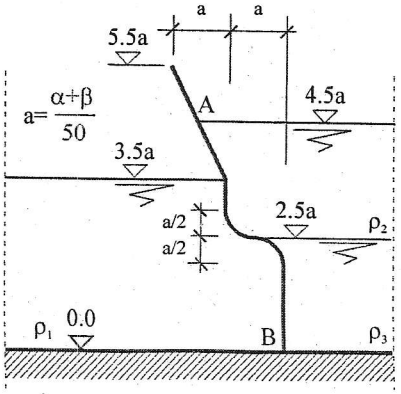
Zanemariti gustinu vazduha.



ZADATAK 2.4

Tri fluida su podeljena zidom širine 3 m, kao na skici. Gustine fluida su $\rho_1=\alpha/3$ kg/dm^3, $\rho_2=1$ kg/dm^3 i $\rho_3=1+\frac{\beta}{40}$ kg/dm^3. Izračunati:

1. pijezometarske kote fluida,
2. ukupnu horizontalnu komponentu hidrostatičke sile (intenzitet i mesto delovanja) na zid A-B ravanskim postupkom (ucrtati silu na skici) od strane sva tri fluida.



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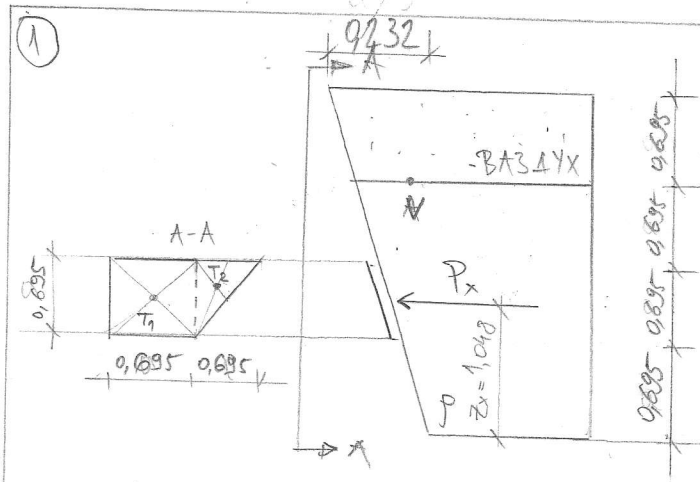


$a = \frac{\alpha+b}{42} = 0,695$

$\rho = 1 + \frac{\alpha}{4} = 2,25 \ kg/dm^3$

$\Pi = 3,2a = 2,224 m$

$p_v = ?$ P_x

$\Pi = \frac{p_v}{\rho g} + z_v \Rightarrow p_v = \rho g (\Pi - z_v)$ $p_v = 2,25 \cdot 10^3 \cdot 9,81 \cdot (2,224 - 2,085)$

$\boxed{p_v = 3,07 kPa}$

$A_{x_1} = 0,695 \cdot 0,695$ $\boxed{A_{x_1} = 0,483 m^2}$

$A_{x_2} = \frac{0,695 \cdot 0,695}{2}$ $\boxed{A_{x_2} = 0,241 m^2}$

$P_x = p_T A_x$

$p_{T_1} = \rho g (\Pi - z_{p_{T_1}}) = 2,25 \cdot 10^3 \cdot 9,81 \cdot (2,224 - 1,0425)$

$\boxed{p_{T_1} = 26,08 kPa}$

$p_{T_2} = \rho g (\Pi - z_{pT_2}) = 2,25 \cdot 10^3 \cdot 9,81 (2,224 - 1,158)$ $\boxed{p_{T_2} = 23,53 kPa}$

$P_{x1} = p_{T_1} \cdot A_{x_1} = 26,08 \cdot 0,483 \Rightarrow \boxed{P_{x_1} = 12,60 kN}$

$P_{x2} = p_{T_2} \cdot A_{x_2} = 23,53 \cdot 0,241 \Rightarrow \boxed{P_{x_2} = 5,67 kN}$

$\left\{ \boxed{P_x = 18,27 kN} \right.$

$I^1_{yy} = \frac{a^4}{12} = \frac{0,695^4}{12} \Rightarrow \boxed{I^1_{yy} = 0,019 m^4}$ $I^2_{yy} = \frac{a^4}{36} = \frac{0,695^4}{36} \Rightarrow \boxed{I^2_{yy} = 0,0065 m^4}$

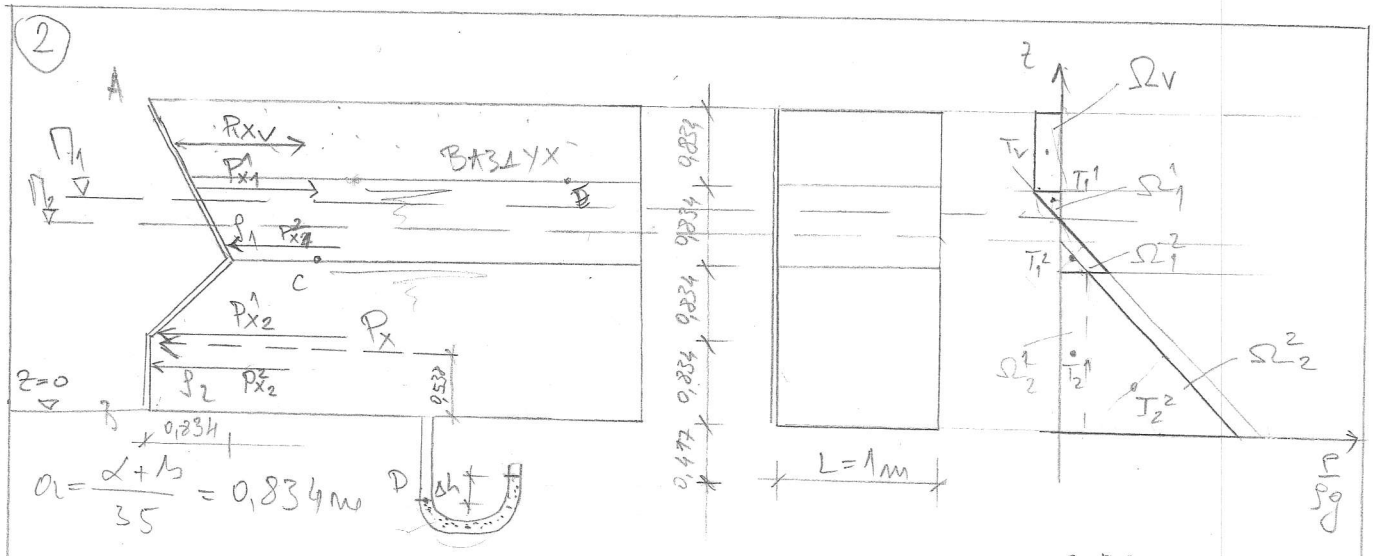
$e^1 = - \frac{\rho g I^1_{yy}}{P_{x1}} \Rightarrow \frac{-2,25 \cdot 10^3 \cdot 9,81 \cdot 0,019}{12,60} \Rightarrow \boxed{e_1 = -0,033 m}$

$e_2 = - \frac{2,25 \cdot 10^3 \cdot 9,81 \cdot 0,0065}{5670} \Rightarrow \boxed{e_2 = -0,025 m}$

$P_x \cdot z_x = P_{x1} \cdot z_{x_1} + P_{x2} \cdot z_{x2} \Rightarrow z_x = \frac{12600 \cdot (0,695 + \frac{0,695}{2} - 0,033) + 5670 \left(0,695 + \frac{2}{3} \cdot 0,695 - 0,025\right)}{18270}$

$\boxed{z_x = 1,048 m}$

(2)



$a_1 = \frac{a+b}{3.5} = 0,834\,m$

$\rho_2 = 1\ kg/dm^3$ $\quad \Pi_1 = 2,7a = 2,252\,m$

$\rho_1 = 0,75a = 0,625\ kg/dm^3$ $\quad \rho_{\check{z}} = 13,6\ kg/dm^3$

$z_V = 3,5a = \underline{2,919\,m}$

$z_1^1 = \Pi_1 + \frac{2}{3}(3 \cdot 0,834 - \Pi_1) = \underline{2,419\,m}$

$z_1^2 = 2a + \frac{1}{3}(\Pi_1 - 2a) = \underline{1,863\,m}$

$z_2^1 = \underline{0,834\,m}$

$z_2^2 = \frac{2}{3}a = \underline{0,556\,m}$

$p_C = \rho_1 g(\Pi_1 - z_C) = 625 \cdot 9,81 \cdot (2,252 - 1,668)$

$\boxed{p_C = 3,58\ kPa}$

$\Pi_2 = \frac{p_C}{\rho_2 g} + z_C = \frac{3,58 \cdot 10^3}{1 \cdot 10^3 \cdot 9,81} + 1,668$

$\boxed{\Pi_2 = 2,033\,m}$

$\rho_2 g(\Pi_2 - z_D) = \rho_{\check{z}} g(\Pi_{\check{z}} - z_D)$

$1000 \cdot 9,81(2,033 - z_D) = 13600 \cdot 9,81(-0,417 - z_D)$

$19943,73 - 9810 z_D = -55634,47 - 133416 z_D$

$123606 z_D = -75578,2$

$\boxed{z_D = -0,612}$

$\Pi_{\check{z}} - \Delta h = -0,612$

$-0,417 - \Delta h = -0,612$

$\boxed{\Delta h = 0,195\,cm}$

$\Omega_1^1 = A_{x_1}^1 = \frac{0,25 \cdot 0,25}{2} = \underline{0,0312\,m^2}$

$\Omega_1^2 = A_{x_1}^2 = \frac{0,584 \cdot 0,584}{2} = \underline{0,171\,m^2}$

$\Omega_2^1 = A_{x_2}^1 = \frac{0,365 \cdot 2 \cdot 0,834}{2} = \underline{0,609\,m^2}$

$\Omega_2^2 = A_{x_2}^2 = \frac{(2 \cdot 0,834)^2}{2} = \underline{1,391\,m^2}$

$\Omega_V = A_V = 0,25 \cdot 0,834 = \underline{0,209\,m^2}$

$p_E = \rho_1 g(\Pi_1 - z_E) = 625 \cdot 9,81 \cdot (2,252 - 2,502)$

$\boxed{p_E = -1532,81\ Pa}$

$P_{x_1}^1 = \rho_1 g z_1^1 \Omega_1^1 = 625 \cdot 9,81 \cdot 1 \cdot 0,0312$

$\boxed{P_{x_1}^1 = -191,295\ N}$

$P_{xV} = \rho_1 g \frac{p_E}{\rho_1 g} \cdot L \cdot \Omega_V$

$P_{x_1}^2 = 625 \cdot 9,81 \cdot 1 \cdot 0,171$

$\boxed{P_{x_1}^2 = 1048,44\ N}$

$\boxed{P_{xV} = -1278,36\ N}$

$z_x = \frac{P_{x_1}^1 z_1^1 + P_{x_1}^2 z_1^2 + P_{x_2}^1 z_2^1 + P_{x_2}^2 z_2^2 + P_{xV} z_V}{P_x}$

$z_x = \frac{-191,295 \cdot 2,419 + 1048,44 \cdot 1,863 + 5974,29 \cdot 0,834 + 13645,71 \cdot 0,556 - 1278,36 \cdot 2,919}{19198,78}$

$P_{x_2}^1 = 1000 \cdot 9,81 \cdot 1 \cdot 0,609$

$\boxed{P_{x_2}^1 = 5974,29\ N}$

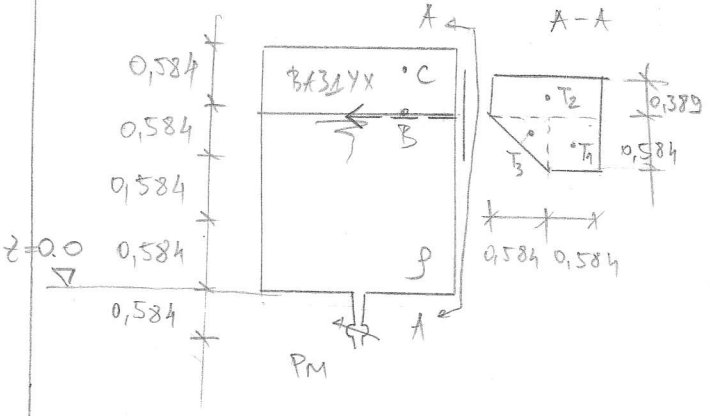
$P_{x_2}^2 = 1000 \cdot 9,81 \cdot 1,391$

$\boxed{P_{x_2}^2 = 13645,71\ N}$

$\boxed{P_x = 19198,78\ N}$

$\boxed{z_x = 0,538\,m}$

(3)



$$a = \frac{\alpha + \beta}{50} = 0{,}584$$

$\rho = 1{,}1\ kg/dm^3 \quad p_M = 4{,}84\ kPa$

$p_V = ?$

$$\frac{p_M}{\rho g} + z_M = \Pi$$

$$\Pi = \frac{4{,}84 \cdot 10^3}{1100 \cdot 9{,}81} - 0{,}584$$

$$\boxed{\Pi = -0{,}169\ m}$$

$p_B = \rho g (\Pi - z_B)$

$p_B = 1100 \cdot 9{,}81 (-0{,}169 - 1{,}752)$

$\boxed{p_B = -20{,}73\ kPa} = p_C \Rightarrow T_2$

$p_{T_1} = \rho g (\Pi - z_{T_1})$

$p_{T_1} = 1100 \cdot 9{,}81 (-0{,}169 - 1{,}46)$

$\boxed{p_{T_1} = -17{,}58\ kPa}$

$p_{T_3} = 1100 \cdot 9{,}81 (-0{,}169 - 1{,}557)$

$\boxed{p_{T_3} = -18{,}62\ kPa}$

$A_{x_1} = 0{,}584^2 = \boxed{0{,}341\ m^2}$

$A_{x_2} = 0{,}389 \cdot 0{,}584 \cdot 2 = \boxed{0{,}454\ m^2}$

$A_{x_3} = \frac{0{,}584^2}{2} = \boxed{0{,}171\ m^2}$

$p_{T_2} = p_B = p_C = \boxed{-20{,}73\ kPa}$

$p_{x_1} = -17{,}58 \cdot 0{,}341 \Rightarrow \boxed{p_{x_1} = -5{,}995\ kN}$

$p_{x_2} = -20{,}73 \cdot 0{,}454 \Rightarrow \boxed{p_{x_2} = -9{,}411\ kN}$

$p_{x_3} = -18{,}62 \cdot 0{,}171 \Rightarrow \boxed{p_{x_3} = -3{,}184\ kN}$

$\boxed{p_x = -18{,}59\ kN}$

$I_{yy_1} = \frac{a^4}{12} = \frac{0{,}584^4}{12} = \boxed{9{,}69 \cdot 10^{-3}\ m^4}$

$I_{yy_3} = \frac{a^4}{36} = \frac{0{,}584^4}{36} = \boxed{3{,}23 \cdot 10^{-3}\ m^4}$

$e_1 = -\frac{\rho g I_{yy_1}}{p_{x_1}} = -\frac{1100 \cdot 9{,}81 \cdot 9{,}69 \cdot 10^{-3}}{-5{,}995\ k 10^3} \Rightarrow \boxed{e_1 = 0{,}017\ m}$

$e_3 = -\frac{1100 \cdot 9{,}81 \cdot 3{,}23 \cdot 10^{-3}}{-3{,}184 \cdot 10^3} \Rightarrow \boxed{e_3 = 0{,}0109\ m}$

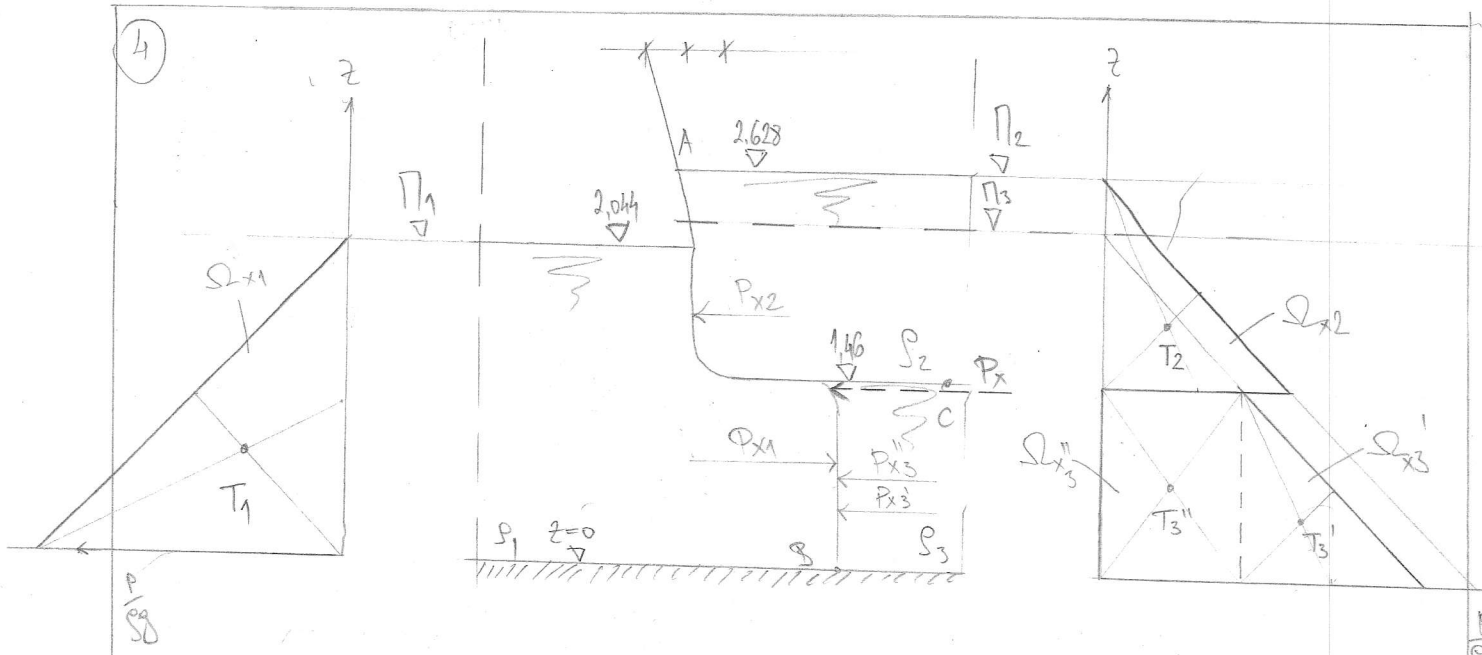
✓

$z_x = \frac{p_{x_1} \cdot z_{x_1} + p_{x_2} \cdot z_{x_2} + p_{x_3} \cdot z_{x_3}}{p_x} = \frac{-5{,}995 \cdot (1{,}46 - 0{,}017) - 9{,}411 \cdot 1{,}946 - 3{,}184 \cdot (1{,}557 - 0{,}0109)}{-18{,}59}$

$\boxed{z_x = 1{,}715\ m}$

✓

4



$a = 0{,}584$ $\quad \rho_1 = 1{,}6\ kg/dm^3$

$L = 3m$ $\quad \rho_2 = 1\ kg/dm^3$

$\rho_3 = 1{,}605\ kg/dm^3$

$z_{T_1} = 0{,}681\ m$

$z_{T_2} = 1{,}849\ m$

$z_{T_3'} = 0{,}487\ m$

$z_{T_3''} = 0{,}73m$

$\Pi_1 = 2{,}044m$

$\Pi_2 = 2{,}628m$

$p_c = \rho_2 g (\Pi_2 - z_c)$

$p_c = 1000 \cdot 9{,}81 (2{,}628 - 1{,}46)$

$p_c = 11{,}46\ kPa$

$\Omega x_1 = \frac{2{,}044^2}{2} = 2{,}089\ m^2$

$\Omega x_2 = \frac{1{,}168^2}{2} = 0{,}682 m^2$

$\Omega x_3' = \frac{1{,}46^2}{2} = 1{,}066 m^2$

$\Omega x_3'' = 0{,}728 \cdot 1{,}46 = 1{,}063\ m^2$

$\Pi_3 = \frac{p_c}{\rho_3 g} + z_c$

$\Pi_3 = \frac{11{,}46 \cdot 10^3}{1605 \cdot 9{,}81} + 1{,}46$

$\Pi_3 = 2{,}188m$

$P_{x_1} = \rho_1 g \Omega x_1 L = 1600 \cdot 9{,}81 \cdot 2{,}089 \cdot 3$

$P_{x_1} = 98366{,}83\ N$

$P_{x_2} = 1000 \cdot 9{,}81 \cdot 0{,}682 \cdot 3$

$P_{x_2} = 20071{,}26\ N$

$P_{x_3'} = 1605 \cdot 9{,}81 \cdot 3 \cdot 1{,}066$

$P_{x_3'} = 50352{,}67\ N$

$P_{x_3''} = 1605 \cdot 9{,}81 \cdot 1{,}063 \cdot 3$

$P_{x_3''} = 50210{,}96\ N$

$P_x = 22268{,}06\ N$

ДЕЛИЌЕ КА КОНТУРИ СА ДЕСНЕ СТРАНЕ

$$z_x = \frac{20071{,}26 \cdot 1{,}849 + 50352{,}67 \cdot 0{,}487 + 50210{,}96 \cdot 0{,}73 - 98366{,}83 \cdot 0{,}681}{22268{,}06}$$

$z_x = 1{,}406\ m$

✓