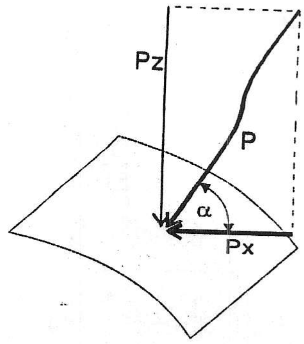
Vežba H2 – ODREĐIVANJE HIDROSTATIČKE SILE NA POVRŠ

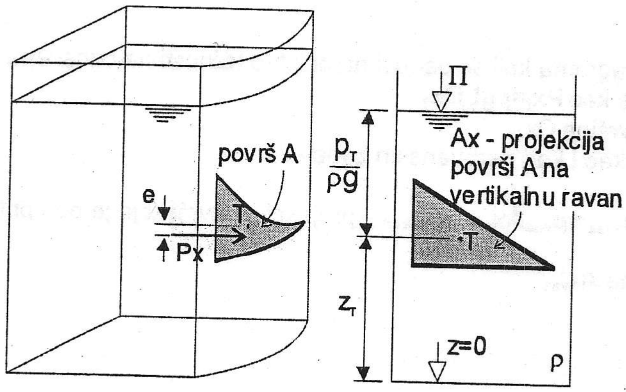
Ukupna sila P na površ A se razlaže na:

- vertikalnu komponentu **Pz** i
- horizontalnu komponentu **Px**.



HORIZONTALNA KOMPONENTA SILE Px

1. Slučaj kada je projekcija površi na vertikalnu ravan (Ax) proizvoljnog oblika



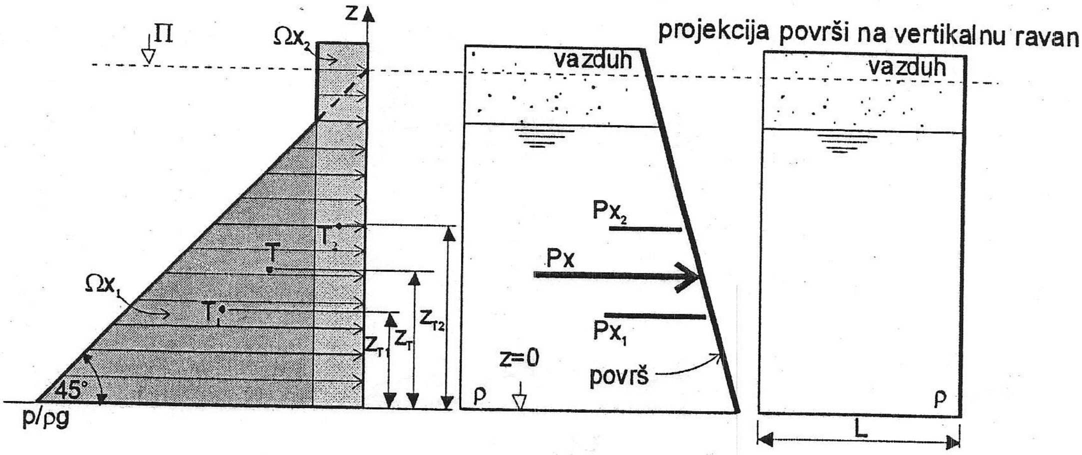
- **Intenzitet sile:** $Px = p_T Ax$
  p_T - pritisak u težištu projekcije Ax: $p_T = \rho g\,(\Pi - z_T)$
- **Mesto delovanja:** pošto se pritisak menja po visini, mesto delovanja sile je
  * ispod težišta (e<0), ukoliko je $p_T>0$
  * iznad težišta (e>0), ukoliko je $p_T<0$

  $$e = \frac{-\rho g \cdot Iyy}{Px}$$

  Iyy – centrifugalni momenat inercije u odnosu na horizontalnu osu koja prolazi kroz težište (vidi prilog u elaboratu)
- **Smer delovanja:**
  * ka površi, ukoliko je $p_T>0$
  * od površi, ukoliko je $p_T<0$.

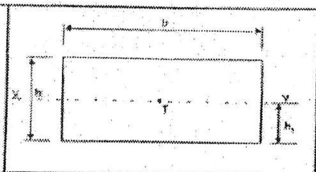
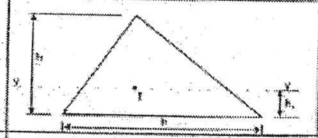
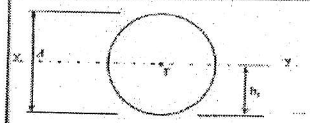
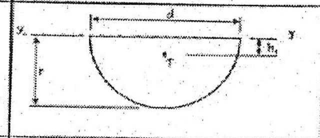
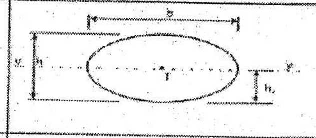
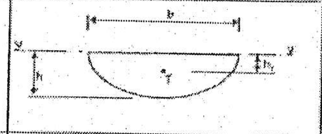
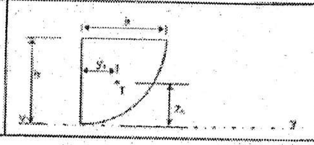
2. Slučaj kada je projekcija površi na vertikalnu ravan Ax pravougaonog oblika – ravanski zadatak

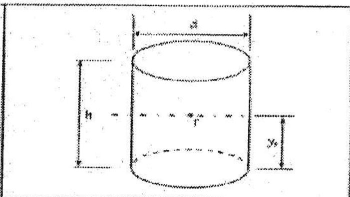
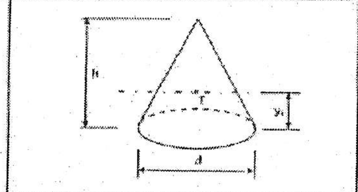
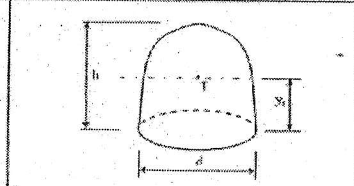
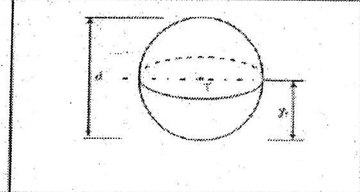
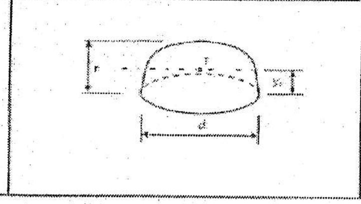
- **Prvi način određivanja horizontalne komponente hidrostatičke sile:**



- nacrta se dijagram visine pritiska (objašnjenje u vežbi H1). Površina dijagrama je Ωx
- površina Ωx se izdeli na delove (Ωx_1, Ωx_2...)
- izračunaju se horizontalne komponente sile po obrascu $\mathbf{Px_i = \rho_i g L \Omega x_i}$ (u primeru: $Px_1 = \rho g L \Omega x_1$ $Px_2 = \rho g L \Omega x_2$)
- mesto delovanja komponenti sile je na kotama težišta površina Ωx_i
- smer delovanja se određuje na isti način kao i kod neravanskih zadataka
- ukupna horizontalna sila dobija se vektorskim sabiranjem komponenti: $\overrightarrow{Px} = \sum \overrightarrow{Px_i}$
- mesto delovanja određuje se preko jednakog momenta oko proizvoljne tačke: $z_{Px} = \frac{\sum Px_i z_{Ti}}{Px}$

$z_x = z_T + e$

		površina	položaj težišta	momenat inercije
PRAVOUGAONIK		$b\,h$	$h_T = \frac{h}{2}$	$I_x = \frac{b\,h^3}{12}$
TROUGAO		$\frac{b\,h}{2}$	$h_T = \frac{h}{3}$	$I_x = \frac{b\,h^3}{36}$
KRUG		$\frac{\pi d^2}{4}$	$h_T = \frac{d}{2}$	$I_x = \frac{\pi d^4}{64}$
POLUKRUG		$\frac{\pi d^2}{8}$	$h_T = \frac{4r}{3\pi}$	$I_x = \frac{\pi d^4}{128}$
ELIPSA		$\frac{\pi\,b\,h}{4}$	$h_T = \frac{h}{2}$	$I_x = \frac{\pi\,b\,h^3}{64}$
POLUELIPSA		$\frac{\pi\,b\,h}{4}$	$h_T = \frac{4h}{3\pi}$	$\frac{\pi b^3 h}{64}$
PARABOLA		$\frac{2}{3}\,b\,h$	$z_T = \frac{3h}{5}$ $y_T = \frac{3b}{8}$	$I_x = \frac{2\,b\,h^3}{7}$

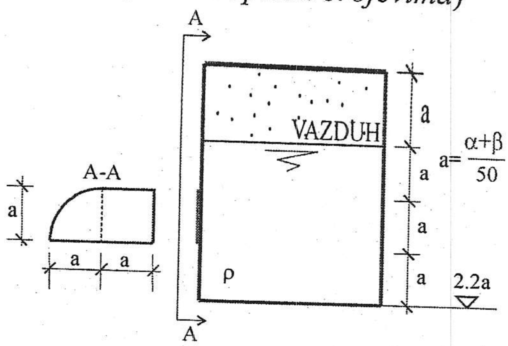
		zapremina	položaj težišta
VALJAK		$\frac{\pi d^2 h}{4}$	$y_T = \frac{h}{2}$
KUPA		$\frac{1}{3}\,\frac{\pi d^2 h}{4}$	$y_T = \frac{h}{4}$
PARABOLOID		$\frac{1}{2}\,\frac{\pi d^2 h}{4}$	$y_T = \frac{h}{3}$
SFERA		$\frac{\pi d^3}{6}$	$y_T = \frac{d}{2}$
POLUSFERA		$\frac{\pi d^3}{12}$	$y_T = \frac{3r}{8}$

JEDNOSTAVNI ZADACI

(Napomena: slike kotirati u metrima, a ne u opštim brojevima)

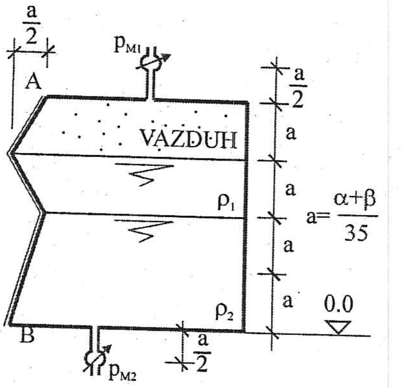
ZADATAK 2.1

U rezervoaru prikazanom na skici nalazi se tečnost gustine ρ=1.2 kg/dm^3. Vazduh iznad slobodne površine tečnosti je pod hidrostatičkim pritiskom $p_{vaz}=\alpha$ kPa. Gustina vazduha se zanemaruje. Izračunati horizontalnu komponentu hidrostatičke sile (intenzitet i mesto delovanja) na poklopac oblika kao na skici (silu ucrtati na skici).



ZADATAK 2.2

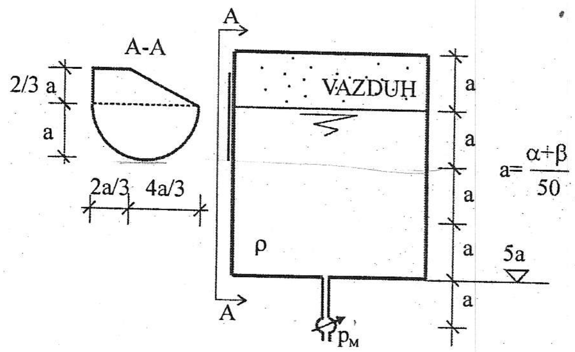
U zatvorenom rezervoaru, širine 1 m, prikazanom na slici nalaze se dva fluida gustina ρ_2=1 kg/dm^3 i $\rho_1=1-\dfrac{\alpha+\beta}{100}$ kg/dm^3 i vazduh pod pritiskom. Gustina vazduha se zanemaruje. Ukoliko je pokazivanje na manometru p_{M2}=1.7β kPa, nacrtati dijagram visine pritisaka i izračunati horizontalnu komponentu hidrostatičke sile na zid A-B (intenzitet i mesto delovanja), ravanskim postupkom (ucrtati silu na skici). Takođe, odrediti čitanje na manometru p_{M1}.



SLOŽENI ZADACI

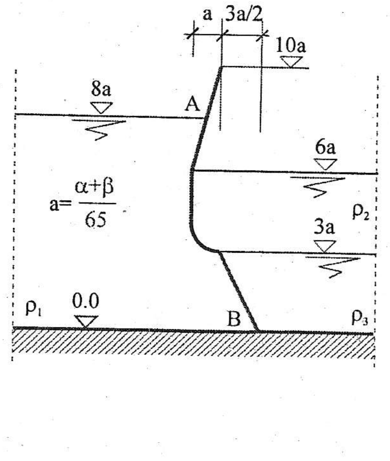
ZADATAK 2.3

U rezervoaru prikazanom na skici nalazi se fluid gustine $\rho=\beta/25$ kg/dm^3. Manometar prikazan na slici pokazuje pritisak od p_M= (2.8α+3.2β) kPa. Gustina vazduha se zanemaruje. Izračunati horizontalnu komponentu hidrostatičke sile (intenzitet i mesto delovanja) na poklopac složene geometrije koji je prikazan na skici (ucrtati silu na skici).



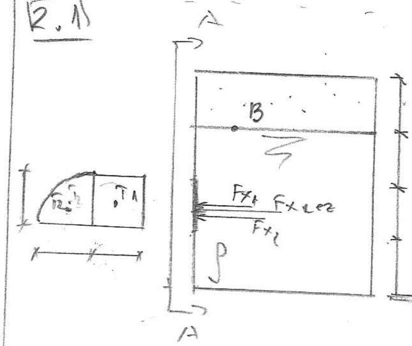
ZADATAK 2.4

Tri fluida gustina ρ_1=a kg/dm^3, ρ_2=1.4 kg/dm^3 i ρ_3 deli zid širine 1.5 m. Izračunati horizontalnu komponentu hidrostatičke sile (intenzitet i mesto delovanja) na zid A-B ravanskim postupkom (ucrtati silu na skici) od strane fluida ρ_1 i ρ_2 sa leve strane. Odrediti potrebnu gustinu fluida ρ_3 tako da ukupna horizontalna sila na zid (od strane sva tri fluida) bude nula.



2.1

$a = \frac{\alpha + \beta}{50} \Rightarrow a = 0{,}444$



$Z_B = 4{,}0248\ m$

$\rho = 1200\ kg/m^3$

$P_{voz} = 6000\ Pa$

$P_B = P_{voz} = 6000\ Pa$

$h_{T2} = \frac{4}{3}\frac{R}{\pi} = \frac{4}{3}\cdot\frac{a}{\pi} = 0{,}328$

$z_{T1} = 1{,}7028 + 0{,}444 + \frac{0{,}444}{2} = 2{,}8638\ m$ $\quad A_1 = a^2 \Rightarrow A_1 = 0{,}599\ m^2$

$z_{T2} = 1{,}7028 + 0{,}444 + 0{,}328 = 2{,}8048\ m$ $\quad A_2 = \frac{a^2\pi}{4} \Rightarrow A_2 = 0{,}471\ m^2$

$\frac{P_B}{\rho \cdot g} + Z_B = \Pi \quad \frac{6000}{1200 \cdot 9{,}81} + 4{,}0248 = \Pi \Rightarrow \Pi = 4{,}534$

$F_x = P_T \cdot A_x$

$P_{T1} = (\Pi - z_{T1})\rho \cdot g \Rightarrow P_{T1} = 19661{,}59\ Pa \Rightarrow F_{x1} = P_{T1} \cdot A_1 \Rightarrow F_{x1} = 11777{,}29\ N$

$P_{T2} = (\Pi - z_{T2})\rho \cdot g \Rightarrow P_{T2} = 20356{,}14\ Pa \Rightarrow F_{x2} = P_{T2} \cdot A_2 \Rightarrow F_{x2} = 9587{,}74\ N$

$I_{yyk} = \frac{bh^3}{12} = \frac{a^4}{12} = 0{,}0299\ m^4$

$I_{yyn} = \frac{\pi \cdot (2 \cdot 0{,}444)^4}{256} - h_T^2 \cdot A_2 = 0{,}0198\ m^4$

$P_{T1} > 0$, $P_{T2} > 0 \Rightarrow$ ka površini

$P_{rez} = F_{x1} + F_{x2} = 21365{,}03$

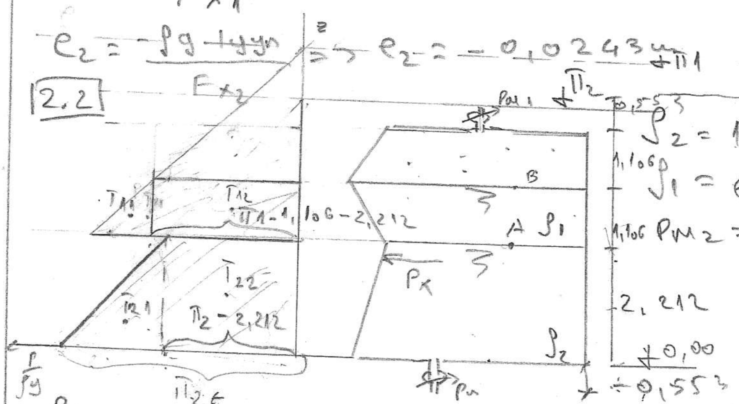
$e_1 = \frac{-\rho g \cdot I_{yyk}}{F_{x1}} \Rightarrow e_1 = -0{,}0299\ m$ $\quad Z_{x1} = Z_{T1} + e_1 \Rightarrow Z_{x1} = 2{,}8339\ m$

$e_2 = \frac{-\rho g \cdot I_{yyn}}{F_{x2}} \Rightarrow e_2 = -0{,}0243\ m$ $\quad Z_{x2} = Z_{T2} + e_2 \Rightarrow Z_{x2} = 2{,}4805\ m$

$Z_{x\,rez} = \frac{F_{x1} \cdot Z_{x1} + F_{x2} \cdot Z_{x2}}{P_{rez}}$

$Z_{x\,rez} = 2{,}8099$

2.2



$L = 1\ m$

$a = 1{,}106$

$A_{vaz} = L \cdot a = 1{,}106\ m^2$

$\frac{P_{M2}}{\rho_2 \cdot g} + Z_{M2} = \Pi_2 \Rightarrow \frac{55590}{1000 \cdot 9{,}81} - 0{,}553 = \Pi_2 \Rightarrow \Pi_2 = 5{,}114\ m$

$\frac{P_A}{\rho_2 \cdot g} + Z_A = \Pi_2 \Rightarrow \frac{P_A}{1000 \cdot 9{,}81} + 2{,}212 = 5{,}114 \Rightarrow P_A = 28468{,}62$

$\frac{P_A}{\rho_1 \cdot g} + Z_A = \Pi_1 \Rightarrow \frac{28468{,}62}{613 \cdot 9{,}81} + 2{,}212 = \Pi_1 \Rightarrow \Pi_1 = 6{,}946\ m$

$Z_{T\,vaz} = 3{,}871\ m$

$Z_{T21} = \frac{1}{3} \cdot 2{,}212 \Rightarrow Z_{T21} = 0{,}737\ m$ $\quad Z_{T22} = 1{,}106\ m$ $\quad Z_{T11} = 2{,}212 + \frac{1{,}106}{3} \Rightarrow Z_{T11} = 2{,}581\ m$ $\quad Z_{T12} = 2{,}765\ m$

$P_{x21} = \rho_2 \cdot g \cdot L \cdot \Omega_{x21} \Rightarrow P_{x21} = 1000 \cdot 9{,}81 \cdot 1 \cdot \frac{2{,}212^2}{2} \Rightarrow P_{x21} = 23999{,}9\ N$

$P_{x22} = \rho_2 \cdot g \cdot L \cdot \Omega_{x22} \Rightarrow P_{x22} = 1000 \cdot 9{,}81 \cdot 1 \cdot 2{,}212 \cdot (5{,}114 - 2{,}212) \Rightarrow P_{x22} = 62972{,}59\ N$

$P_{x11} = \rho_1 \cdot g \cdot L \cdot \Omega_{x11} \Rightarrow P_{x11} = 613 \cdot 9{,}81 \cdot 1 \cdot \frac{1{,}106^2}{2} \Rightarrow P_{x11} = 3677{,}98\ N$

$P_{x12} = \rho_1 \cdot g \cdot L \cdot \Omega_{x12} \Rightarrow P_{x12} = 613 \cdot 9{,}81 \cdot 1 \cdot 1{,}106 \cdot (6{,}946 - 3{,}106) \Rightarrow P_{x12} = 24129{,}698\ N$

$\vec{P_x} = \sum \vec{P_{xi}} \Rightarrow \vec{P_x} = P_{x21} + P_{x22} + P_{x11} + P_{x12} + P_{x\,vaz} =$

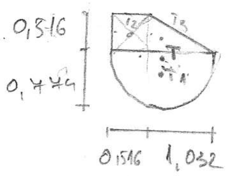
$\frac{P_B}{\rho_1 g} + Z_B = \Pi_1 \Rightarrow P_B = 21817{,}087\ Pa \Rightarrow P_{M1} = P_B = 21817{,}087\ Pa$

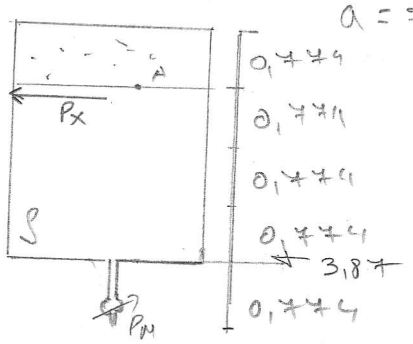
$P_{x\,voz} = P_{voz} \cdot A_{vaz} = 21817{,}032 \cdot 1{,}106 \cdot 1 = 24129{,}64\ N$

$\vec{P_x} = 138909{,}81\ N$

$Z_{Px} = \frac{\vec{P_{x11}} \cdot Z_{T11} + \vec{P_{x12}} \cdot Z_{T12} + \vec{P_{x21}} \cdot Z_{T21} + \vec{P_{x22}} \cdot Z_{T22} + \vec{P_{x\,vaz}} \cdot Z_{T\,voz}}{\vec{P_x}} \Rightarrow Z_{Px} = 1{,}85\ m$

2.3





$$a = \frac{1+\beta}{50} = 0{,}774$$

$$\rho = 1308\ kg/m^3$$

$$P_M = 121440\ Pa$$

$$\frac{P_M}{\rho g} + z_M = \Pi \Rightarrow \Pi = 12{,}56\ m$$

$$A_2 = 0{,}516^2 \Rightarrow A_2 = 0{,}266\ m^2$$

$$A_3 = \frac{0{,}516 \cdot 1{,}032}{2} \Rightarrow A_3 = 0{,}266$$

$$A_1 = \frac{0{,}774^2 \pi}{2}$$

$$A_1 = 0{,}941\ m^2$$

$$P_A = \rho \cdot g \cdot (\Pi - z_A) \Rightarrow P_A = 81710{,}86\ Pa = P_{raz}$$

$$z_{T1} = 3{,}87 + 0{,}774 \cdot 3 - \frac{4 \cdot 0{,}774}{3\pi} \Rightarrow z_{T1} = 5{,}86\ m$$

$$P_{T1} = \rho g(\Pi - z_{T1}) \Rightarrow P_{T1} = 85970{,}92\ Pa$$

0,148

$$P_{x1} = P_{T1} \cdot A_1 \Rightarrow P_{x1} = 80900{,}9\ N$$

$$z_{T2} = 3{,}87 + 3 \cdot 0{,}774 + \frac{0{,}516}{2} \Rightarrow z_{T2} = 6{,}45\ m$$

$$z_{T3} = 3{,}87 + 3 \cdot 0{,}774 + \frac{0{,}516}{3} \Rightarrow z_{T3} = 6{,}36\ m$$

$$I_{yy} = \frac{\pi \cdot (2 \cdot 0{,}774)^4}{128} - \left(\frac{4 \cdot 0{,}774}{3\pi}\right)^2 \cdot 0{,}941 \Rightarrow I_{yy} = 0{,}0394\ m^4$$

$$e = \frac{-\rho g I_{yy}}{P_{T1}} \Rightarrow e = -5{,}88 \cdot 10^{-3}$$

$$z_{x1} = z_{T1} + e \Rightarrow z_{x1} = 5{,}85\ m$$

$$P_{x2} = P_{raz} \cdot A_2 \Rightarrow P_{x2} = 21735{,}09\ N$$

$$P_{x3} = P_{raz} \cdot A_3 \Rightarrow P_{x3} = 21735{,}09\ N$$

$$P_x = P_{x1} + P_{x2} + P_{x3} \Rightarrow P_x = 124371{,}08\ N$$

$$z_x = \frac{P_{x1} \cdot z_{x1} + P_{x2} \cdot z_{T2} + P_{x3} \cdot z_{T3}}{P_x}$$

$$z_x = \frac{751696{,}47}{124371{,}08} \Rightarrow z_x = 6{,}044\ m$$

2.4

$L = 1{,}5\,m$

$\rho_1 = 595\ kg/m^3$

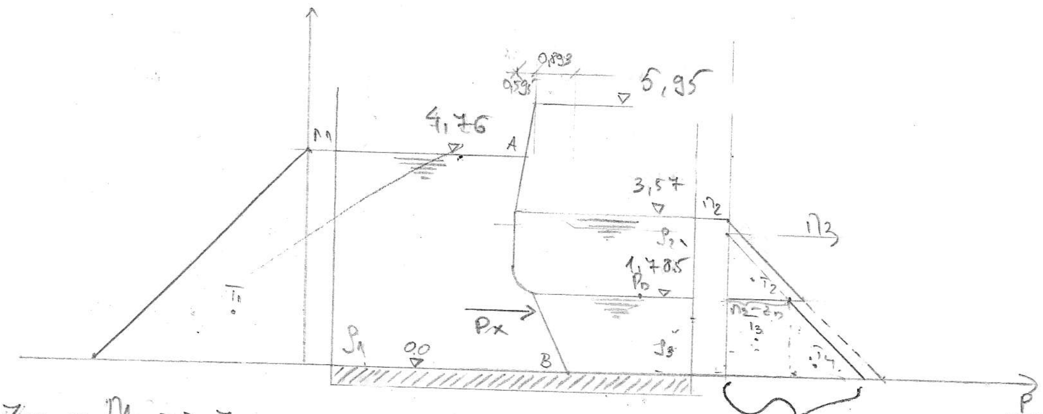
$\rho_2 = 1400\ kg/m^3$

$h_1 = 4{,}76\ m$

$h_2 = 3{,}57\ m$

$a = \frac{32{,}7+6}{65}$

$a = 0{,}595\ m$



$z_{T1} = \frac{h_1}{3} \Rightarrow z_{T1} = 1{,}59\,m$

$z_{T2} = 1{,}785 + \frac{1{,}785}{3} \Rightarrow z_{T2} = 2{,}38\,m$

$\Omega_{x1} = \frac{4{,}76^2}{2} \Rightarrow \Omega_{x1} = 11{,}33\,m^2$

$P_{x1} = \rho_1 g L \Omega_{x1} \Rightarrow P_{x1} = 99198{,}96\ N$

$\Omega_{x2} = \frac{1{,}785^2}{2} \Rightarrow \Omega_{x2} = 1{,}59\,m^2$

$P_{x2} = \rho_2 g L \Omega_{x2} \Rightarrow P_{x2} = 32755{,}59\ N$

$\frac{p_D}{\rho_2 \cdot g} + z_D = h_2 \Rightarrow p_D = 24515{,}19\ Pa$

Уташки D:

p_D - const

g - const

ρ се разликује

$\rho_2 < \rho_3$ јер је изнад

$\frac{p_D}{\rho_2 g} > \frac{p_D}{\rho_3 g}$

$\frac{p_D}{\rho_3 g} + z_D = h_3$

$\rho_3 = \frac{p_D}{(h_3 - z_D)\,g}$

$\Omega_{x3} = z_D \cdot (h_3 - z_D)$

$P_{x3} = \rho_3 \cdot g \cdot L \cdot \Omega_{x3}$

$P_{x3} = \frac{p_D}{(h_3 - z_D) \cdot g} \cdot g \cdot L \cdot z_D (h_3 - z_D) \Rightarrow P_{x3} = p_D \cdot L \cdot z_D \Rightarrow P_{x3} = 65639{,}4\,N$

$P_{x1} = P_{x2} + P_{x3} + P_{x4} \Rightarrow P_{x4} = 803{,}97\ N$

$P_{x4} = \rho_3\, g \cdot L \cdot \Omega_{x4} \quad \left(\Omega_{x4} = \frac{1{,}785^2}{2} = 1{,}59\,m^2\right)$

$\rho_3 = \frac{P_{x4}}{g L \Omega_{x4}} \Rightarrow \rho_3 = \frac{803{,}97}{9{,}81 \cdot 1{,}5 \cdot 1{,}59} \Rightarrow \rho_3 = 34{,}36\ kg/m^3$

$P_x = P_{x1} - P_{x2} \Rightarrow P_x = 66443{,}37\ N$

$z_{Px} = \frac{P_{x1} \cdot z_{T1} + (-P_{x2}) \cdot z_{T2}}{P_x} \Rightarrow z_{Px} = 1{,}2\ m$