



# 9th International Conference on Urban Drainage Modelling Belgrade 2012

## GLOBAL SENSITIVITY ANALYSIS FOR URBAN WATER QUALITY MODELLING: COMPARISON OF DIFFERENT METHODS

**Giorgio Mannina, Alida Cosenza, Gaspare Viviani,  
Peter Vanrolleghem and Marc Neumann**



*Università di Palermo*  
*Dipartimento di Ingegneria Civile,*  
*Ambientale, dei Materiali*



# Introduction

Only few studies in urban drainage deal with GSA

Previous GSA studies mainly focused on other research fields

Limited transferability of the knowledge derived from other fields

There is a need to experience with GSA methods in urban drainage  
to give answers to:



What is the best method? Is any??

What are the main features of each method?

How similar are the results?

To what extend do results differ changing method?



Università di Palermo

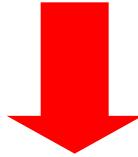
Dipartimento di Ingegneria Civile, Ambientale,  
Aerospaziale, dei Materiali



# Aims of the study

Gain insights about the features of three GSA methods:

- SRC
- Morris screening
- E-Fast



Urban drainage water quality model  
Case study  
Comparison between the results of each method

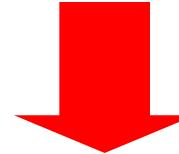


## Terminology



# Introduction: Problem statement

A complete, Clear, Generally Accepted Definition  
Is lacking!!



Suggest a common terminology on the basis of previous studies

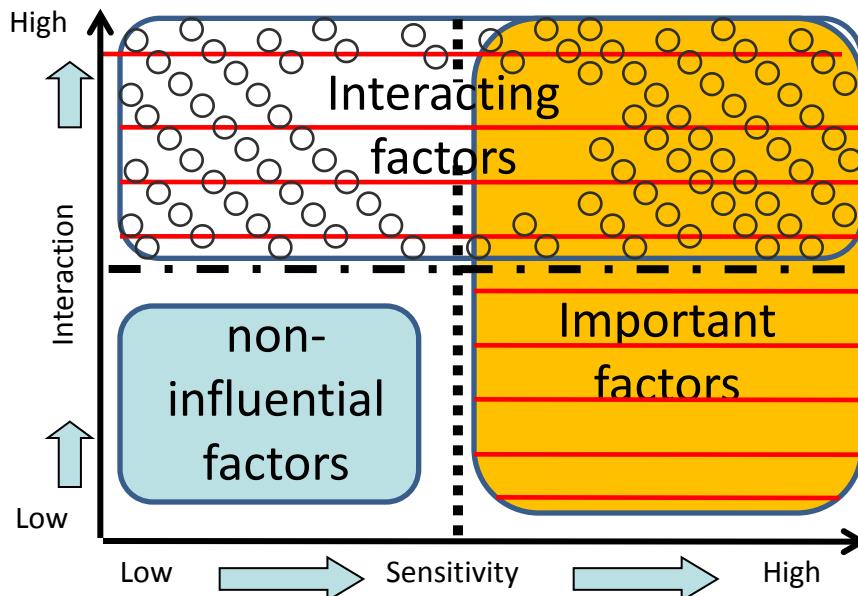
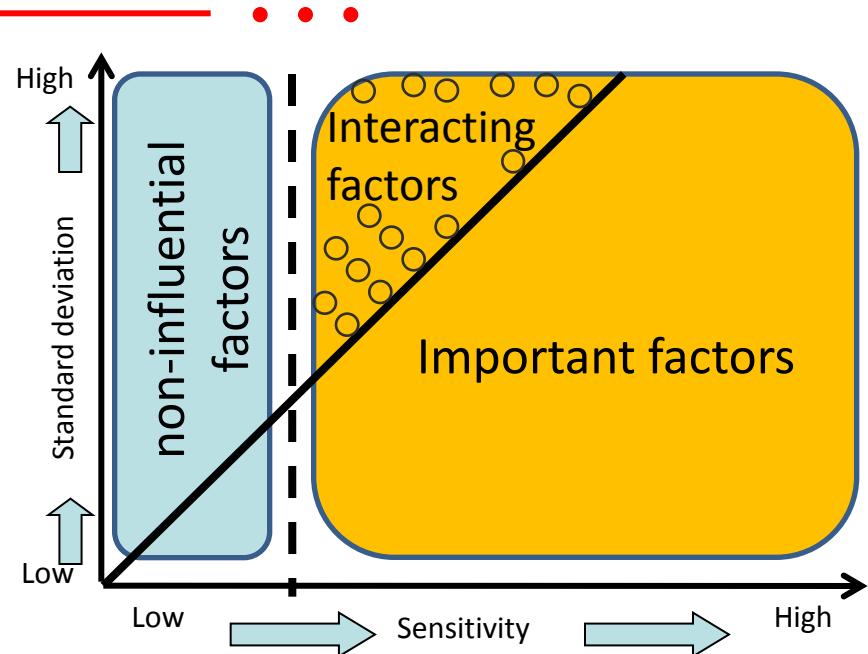
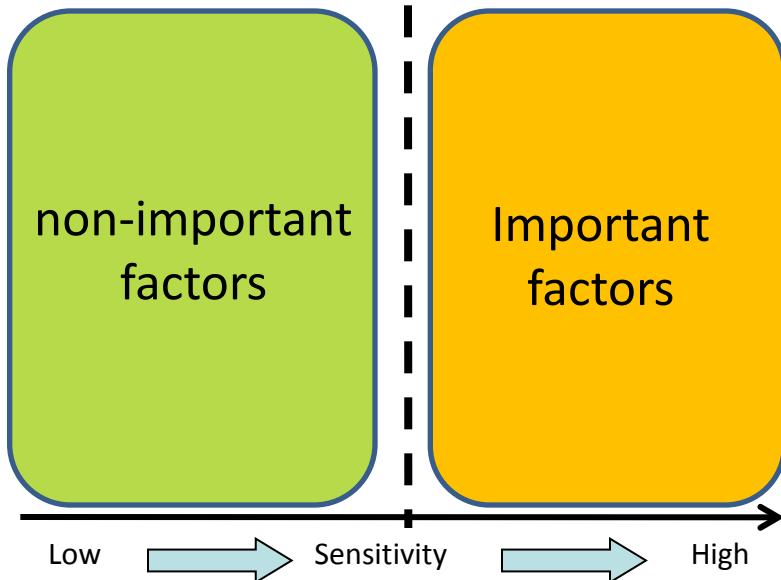


*Università di Palermo*

*Dipartimento di Ingegneria Civile, Ambientale,  
Aerospaziale, dei Materiali*



# Sensitivity methods: making a framework



Influential  
Factors

Factors may include both model inputs and model parameters

# Methods



*Università di Palermo*  
*Dipartimento di Ingegneria Civile, Ambientale,*  
*Aerospaziale, dei Materiali*



Consists of running a Monte Carlo simulation and multivariate linear regression between the model outputs and inputs

$$SRC(x_i) = \beta_i = b_i \cdot \sigma_{x_i} / \sigma_y$$

standardised regression slopes

$\sigma_{x_i}$  and  $\sigma_y$  represent respectively the factor and the model output standard deviation and  $b_i$  the regression slope

**SRCS are valid measures of sensitivity when  $R^2 > 0.7$  (Saltelli et al., 2004)!!!**

$$\sum (\beta_i) = 1 \quad \text{Linear model}$$

The sign of  $\beta_i$  indicates its positive (sign +) or negative (sign -) effect



Università di Palermo

Dipartimento di Ingegneria Civile, Ambientale,  
Aerospaziale, dei Materiali



It is based on a **one-at-a-time (OAT)** perturbation of the model input factors

Factor space is partitioned into **p levels**

OAT is repeated **r times**, computing the **Elementary Effects (EEs)** for the model output considering a **perturbation  $\Delta$** :

$$EEs(\chi_i) = \frac{y(x_1, \dots, x_{i-1}, x_i + \Delta, x_{i+1}, \dots, x_k) - y(x_1, \dots, x_{i-1}, x_i, x_{i+1}, \dots, x_k)}{\Delta}$$

Sensitivity is described by the **absolute mean ( $\mu^*$ )** and **standard deviation ( $\sigma$ )** of the **cumulative distribution** of the  $r^*EEs$



The Extended-FAST is based on the variance decomposition theorem which states that the total variance of the model output ( $\text{Var}(Y)$ ) may be decomposed into conditional variances

$$S_i = \frac{\text{Var}_{xi} (E_{x_{-i}} (Y|x_i))}{\text{Var}(Y)}$$

First order index

Show by which percentage the variance would decrease if the corresponding factor was known

$$S_{Ti} = 1 - \frac{\text{Var}_{x_{-i}} (E_{x_i} (Y|x_{-i}))}{\text{Var}(Y)}$$

Total effect index

Show by which amount the variance would be reduced if all factors except for the  $i$ th were known



# Urban drainage model & Case study



Università di Palermo  
Dipartimento di Ingegneria Civile, Ambientale,  
Aerospaziale, dei Materiali



## Quantity Module

Hydrographs at the inlet  
and at the outlet of the  
sewer system

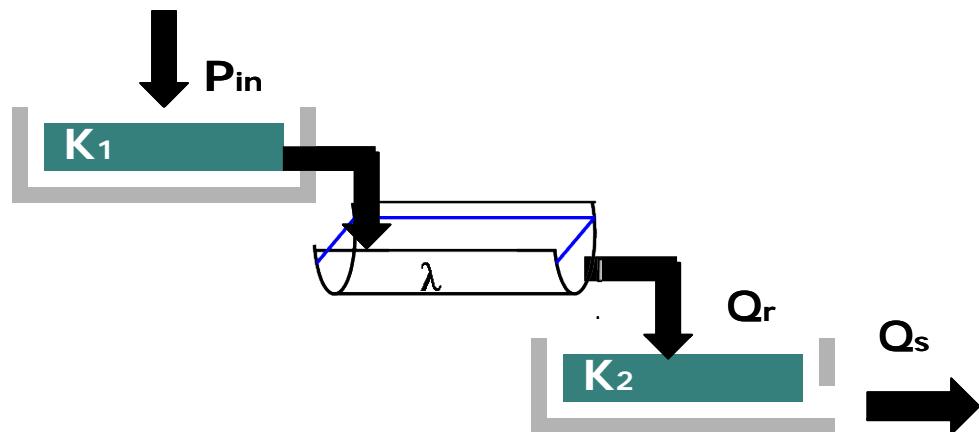
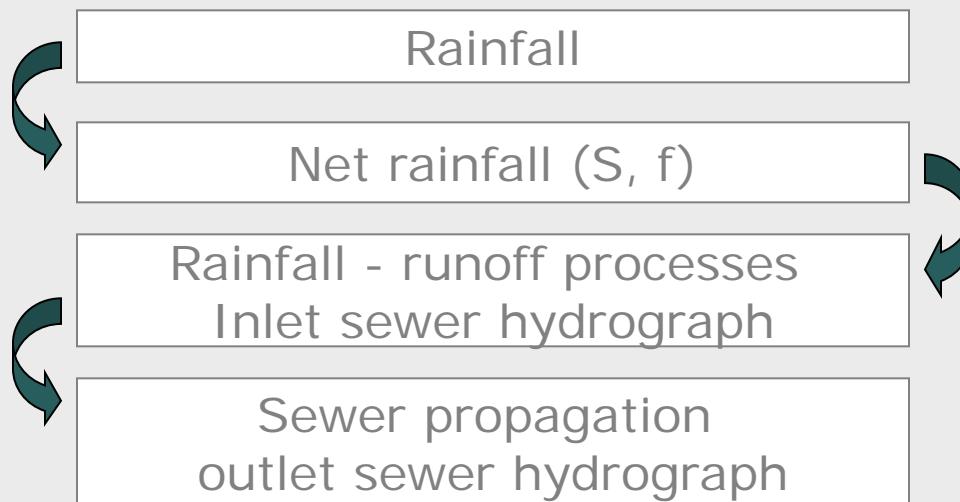
## Quality Module

Pollutographs at the inlet  
and at the outlet of the  
sewer system



# Methods: Urban water quality model (Mannina&Viviani 2010)

## Quantity module



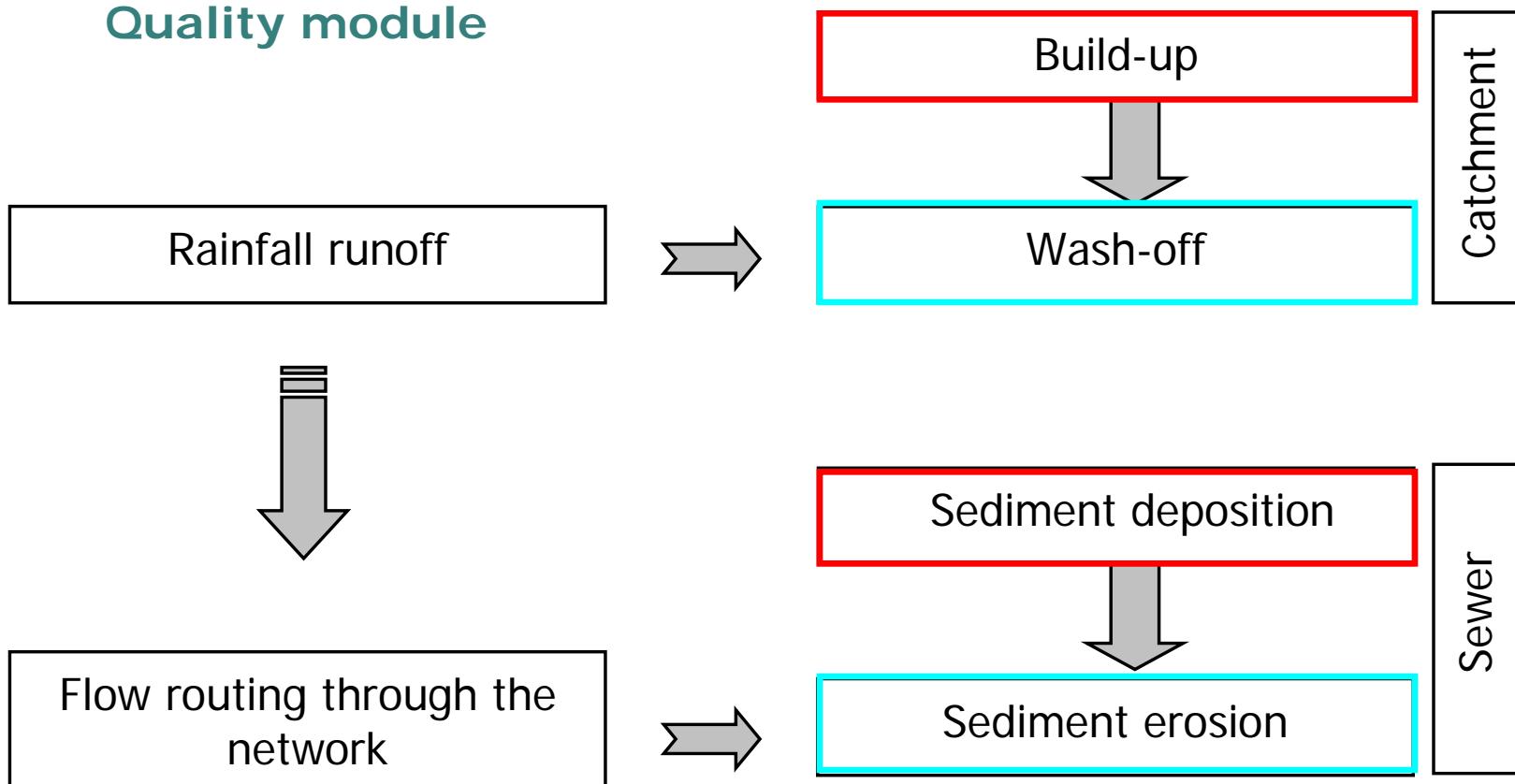
The cascade of two linear reservoirs in series and a linear channel allow to split the hydraulic **phenomena** in the **catchment** from those in the **sewer system**.



# Methods: Urban water quality model (Mannina&Viviani 2010)

## Sewer System model:

### Quality module



# Model application: the case study

## The experimental catchment Montelepre Catchment (IT)



- 36 storm events during one year 2008
- Drained area 40 ha, with an impervious percentage of 75%
- The drainage network ends in a polycentric section pipe 144 cm high and 180 cm wide



# Methods: Application and criteria for comparison

- The model run: 1 year series - 36 events
- Seven model outputs:

- Maximum sewer flow rate ( $Q_{MAX}$ )
- Total sewer flow volume ( $V_{TOT}$ )
- Maximum TSS sewer concentration ( $C_{MAX,TSS}$ )
- Maximum BOD concentration ( $C_{MAX,BOD}$ )
- TSS sewer load ( $L_{TOT,TSS}$ )
- Average TSS sewer concentration ( $C_{AVERAGE,TSS}$ )
- Average BOD sewer concentration ( $C_{AVERAGE,BOD}$ )

- Seventeen model input factors
- Quantity and quality input factors changed simultaneously for each MC
- Uniform distribution – input factor



# Methods: Application and criteria for comparison

- For the SRC and Morris screening:  $CFT = 0.1$
- For E-FAST:  $CFT1 = 0.01$  [correspondence between  $\beta_i^2$  and  $S_i$  (Saltelli et al., 2000)]
- For E-FAST:  $CFT2 = 0.1$  for the value of the interaction  $S_{Ti} - S_i$
- For each method a rank of importance according to factors prioritisation

## Comparison for factors prioritisation:

- $\beta_i^2$  and  $S_i$  SRC vs E-FAST;
- $\beta_i^2$  and  $\mu^*$  SRC vs Morris Screening;
- $\mu^*$  and  $S_i$  Morris Screening vs E-FAST;

## Comparison for factors fixing (Morris screening vs E-FAST):

- $\mu^*$  versus  $S_{Ti}$
- $\sigma$  versus  $S_{Ti}$



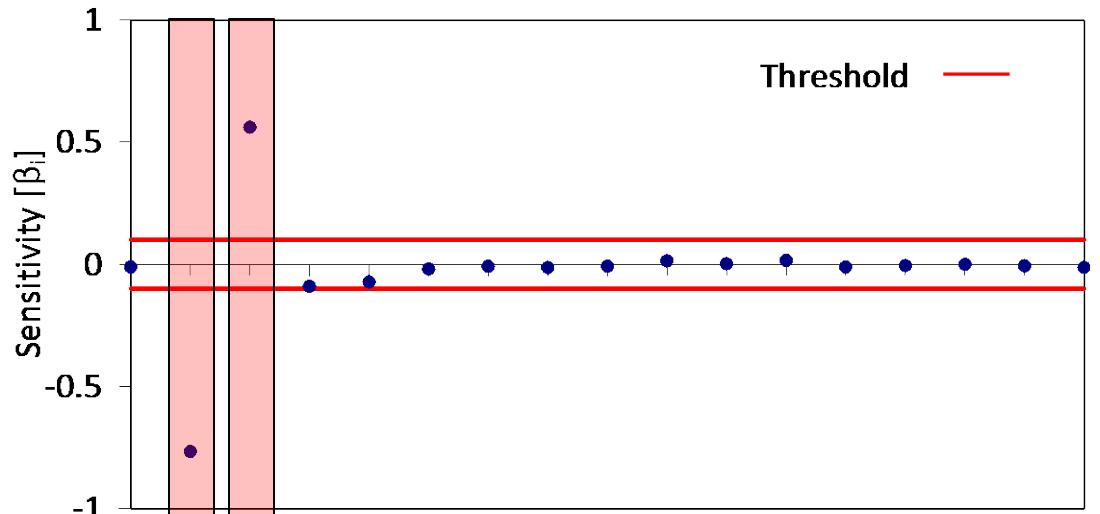
Università di Palermo

Dipartimento di Ingegneria Civile, Ambientale,  
Aerospaziale, dei Materiali



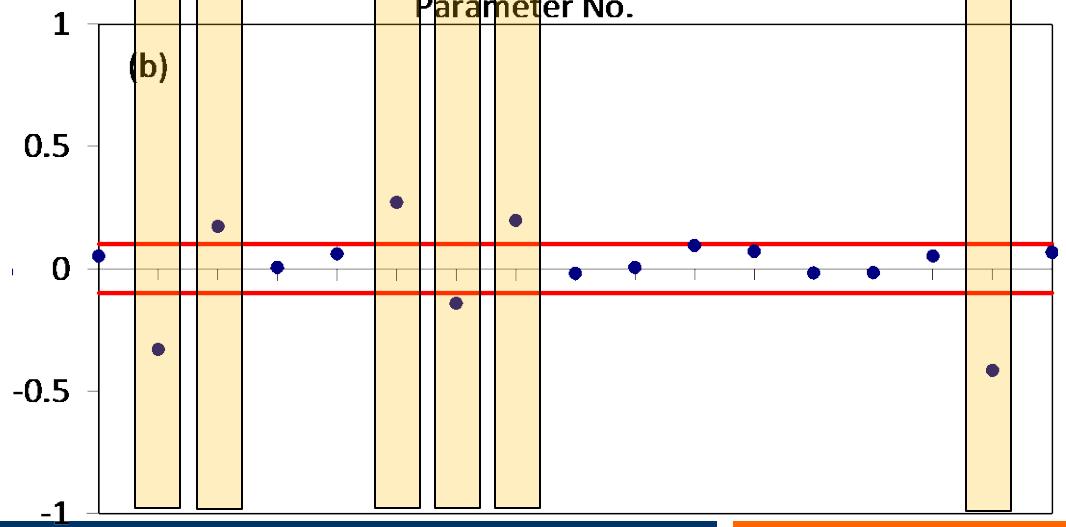
# Results: SRC ( $Q_{MAX} - C_{MAX,BOD}$ )

Quantity



Threshold

Quality



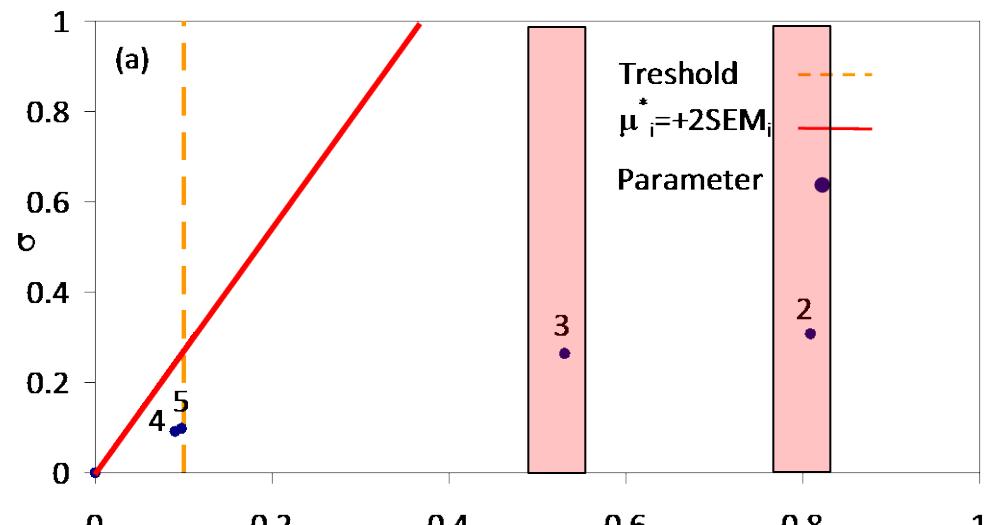
Università di Palermo

Dipartimento di Ingegneria Civile, Ambientale,  
Aerospaziale, dei Materiali

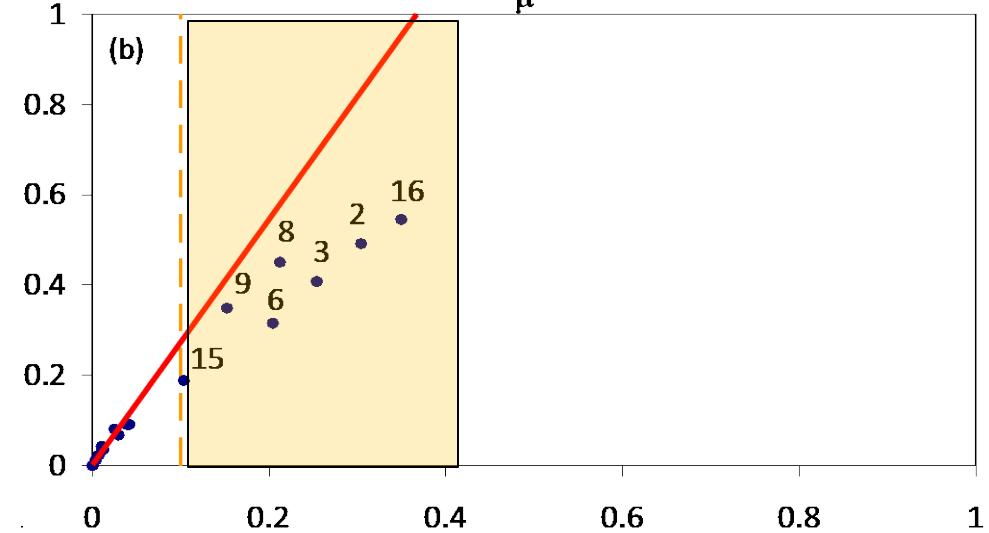


# Results: Morris screening ( $Q_{MAX} - C_{MAX,BOD}$ )

Quantity



Quality



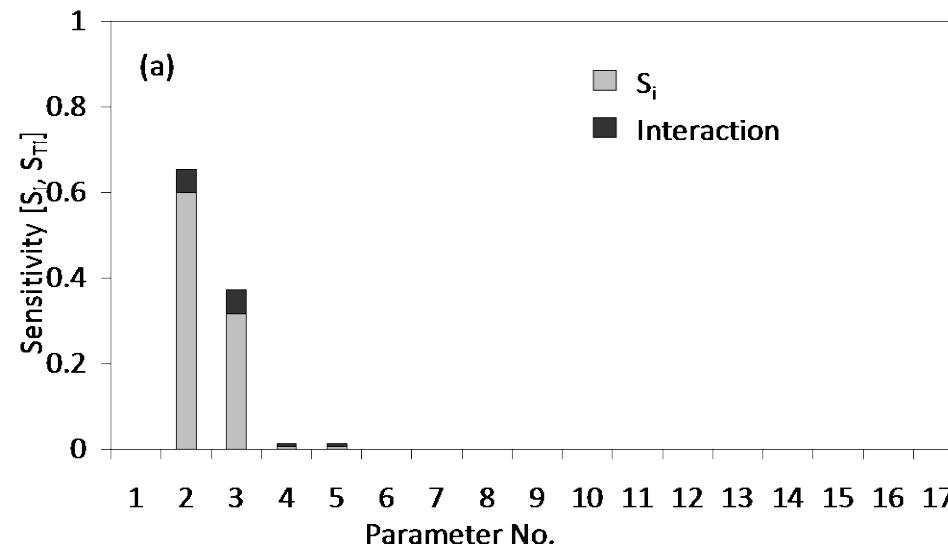
Università di Palermo

Dipartimento di Ingegneria Civile, Ambientale,  
Aerospaziale, dei Materiali

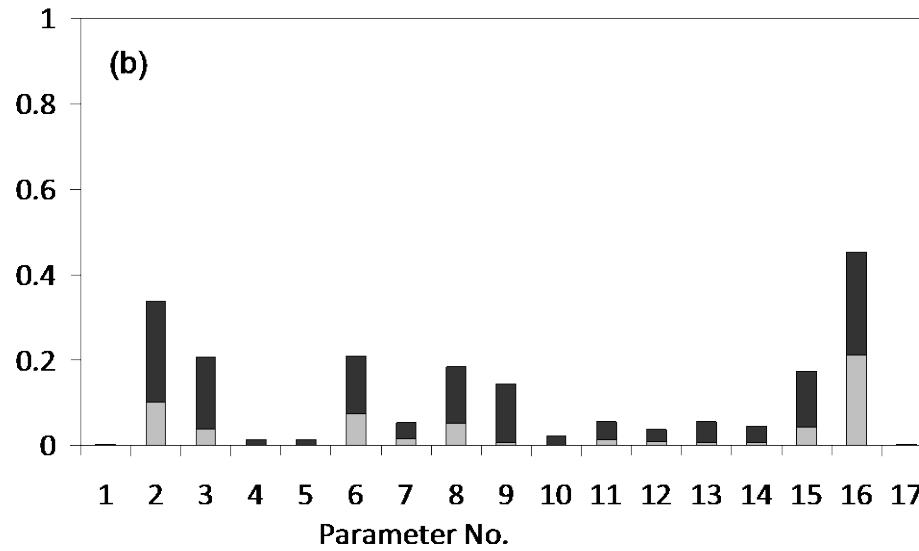


# Results: E-FAST ( $Q_{MAX} - C_{MAX,BOD}$ )

Quantity



Quality

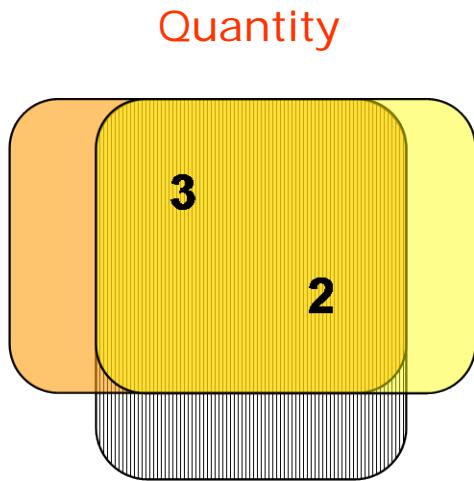


Università di Palermo

Dipartimento di Ingegneria Civile, Ambientale,  
Aerospaziale, dei Materiali



# Results: comparison among the methods



Factors Prioritisation

Important Factors



SRC

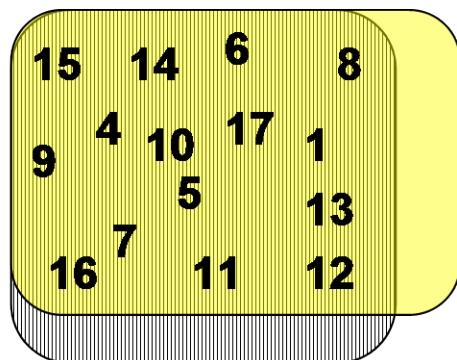


Morris screening



E-FAST

Quantity



Factors Fixing

Non-Influential



Morris screening



E-FAST



Università di Palermo

Dipartimento di Ingegneria Civile, Ambientale,  
Aerospaziale, dei Materiali



# Conclusions

---

- A comparison between three GSA methods: **important, non-influential and interacting** model input factors of an urban stormwater model;
- SRC method is **inside its range of applicability** for the outputs  $Q_{MAX}$ ,  $V_{TOT}$  and  $L_{TOT,TSS}$  and outside for  $C_{MAX,TSS}$ ,  $C_{MAX,BOD}$ ,  $C_{AVERAGE,TSS}$  and  $C_{AVERAGE,BOD}$ .
- The quality sub-module is **non-linear**: attention to non-influential factors.
- **E-FAST** allowed the **quantification of the interactions** of model factors.
- In terms of **factor fixing** similar results were obtained between Morris screening and E-FAST methods for  $Q_{MAX}$ .
- For  $C_{MAX,BOD}$  the Morris screening showed **some differences** (semi-quantitative method!).



**Reminder:**

**Give your preference for:**

**The Best Young Researcher Presentation**



**Did you enjoy??**



**If YES...give your preference:  
Giorgio Mannina**



# 9th International Conference on Urban Drainage Modelling Belgrade 2012

**THANK YOU FOR YOUR ATTENTION!**

[giorgio.mannina@unipa.it](mailto:giorgio.mannina@unipa.it)



*Università di Palermo*  
*Dipartimento di Ingegneria Civile,*  
*Ambientale, dei Materiali*

