State-space calibration of radar rainfall and stochastic flow forecasting for use in real-time Control of urban drainage systems

9th Int. Conf. On Urban Drainage Modeling

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Methodology

- Adjust radar measurements to raingauge data using state-space models with different layouts
- Create flow predictions for 2 catchments using stochastic greybox models using different rainfall inputs
- Use skill scores to evaluate which rainfall input results in the best stochastic flow predictions

Study Area and Data

- 5min flow observations
- 5min rain gauge observations
- 10min C-band radar data
- Period 25/06-29/09/2010

Radar – Rain Gauge Merging – State Space Model

$$\underline{X}_t = \underline{f}\left(\underline{X}_{t-1}, \underline{u}_{t-1}\right) + \underline{e}_{1,t}$$

System Equation:

Predict merged rainfall in pixels

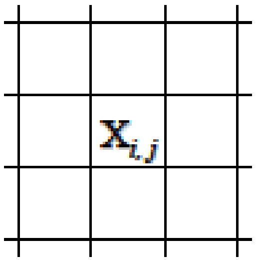
$$\underline{Y}_t = \underline{h}\left(\underline{X}_t\right) + \underline{e}_{2,t}$$

Observation Equation:

Relate model predictions to observations

From Grum, Harremoës, Linde (2002): Assimilating a multitude of rainfall and runoff data using a stochastic state space modeling approach. 9th ICUD, Portland, Oregon USA, 2002

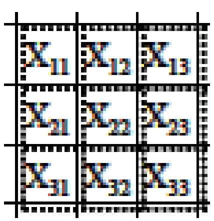
Radar – Rain Gauge Merging – State Space Model



$$X_{i,j}(t) = \sum_{k=-1}^{1} \sum_{l=-1}^{1} \alpha_{k,l} \cdot X_{i+k,j+l}(t-\Delta t) + e_{Xi,j}$$

System Equation:

Predict merged rainfall in pixels



radar rain intensity readings — error

$$\begin{bmatrix} \vdots \\ Y_{Radar\,11} \\ Y_{Radar\,12} \\ Y_{Radar\,13} \\ Y_{Radar\,21} \\ Y_{Radar\,22} \\ Y_{Radar\,23} \\ Y_{Radar\,31} \\ Y_{Radar\,32} \\ Y_{Radar\,33} \\ \vdots \end{bmatrix}_t = \begin{bmatrix} \vdots & & & & & & & & \vdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \vdots & & & & & & & & \vdots \end{bmatrix} \begin{bmatrix} X_{11} \\ X_{12} \\ X_{13} \\ X_{21} \\ X_{22} \\ X_{23} \\ X_{31} \\ X_{32} \\ X_{33} \end{bmatrix}_t + \begin{bmatrix} \vdots \\ e_{Radar\,11} \\ e_{Radar\,12} \\ e_{Radar\,13} \\ e_{Radar\,21} \\ e_{Radar\,22} \\ e_{Radar\,23} \\ e_{Radar\,31} \\ e_{Radar\,32} \\ e_{Radar\,33} \\ \vdots \end{bmatrix}_t$$

Observation Equation:

Relate model predictions to observations

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Radar – Rain Gauge Merging – State Space Model

- Parameter Estimation: Maximum Likelihood
- Model Layouts with different observation covariance structures:
  - Model 1 – 1 variance for all radar observations & 1 variance for all rain gauge observations
  - Model 2 – consider correlation between radar pixels (estimated from variogram)
  - Model 3 – as model 1, but include error marker for missing radar observations

Stochastic Flow Forecasting

Stochastic Greybox Model

$$d\begin{bmatrix} S_{1,t} \\ S_{2,t} \end{bmatrix} = \begin{bmatrix} A \cdot P + a_0 - \frac{1}{K} S_{1,t} \\ \frac{1}{K} S_{1,t} - \frac{1}{K} S_{2,t} \end{bmatrix} dt + \begin{bmatrix} \sigma(S_{1,t}) \\ \sigma(S_{2,t}) \end{bmatrix} d\omega_t$$

System Equation

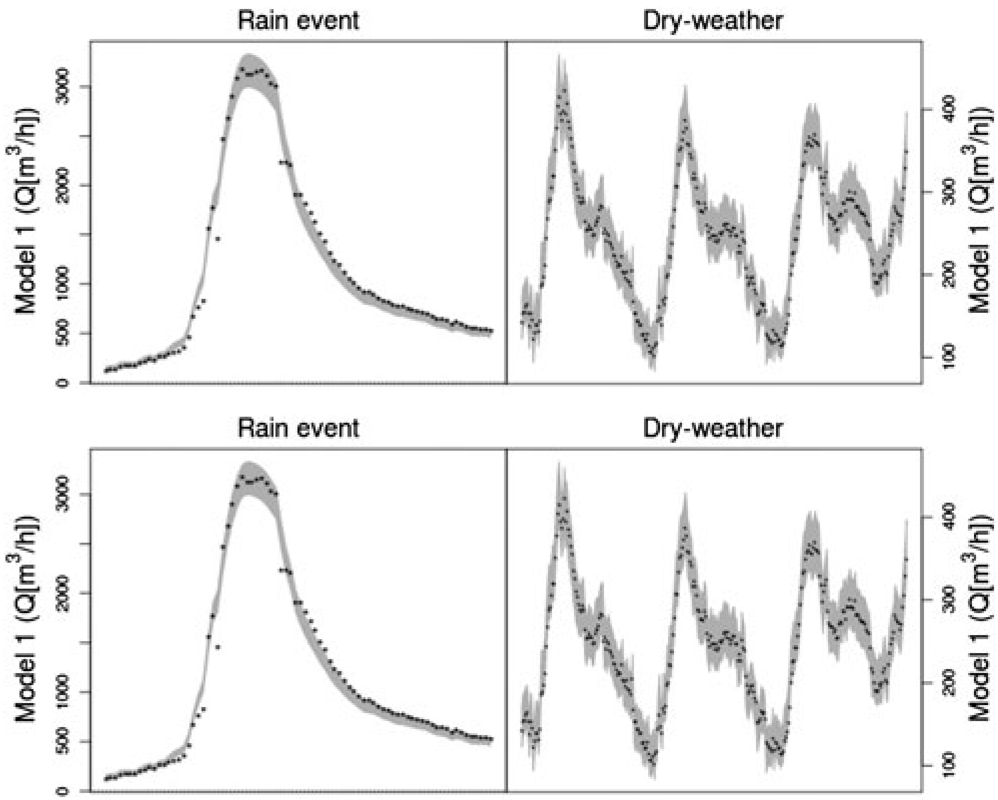
$$\log(Q_k) = \log\left(\frac{1}{K} S_{2,k} + D_k \right) + e_k$$

Observation Equation

Modeling framework: CTSM – continuous time stochastic modeling (open source), as package for R

see Breinholt A., Thordarson F.Ø., Møller J.K., Grum M., Mikkelsen P.S., Madsen H., “Grey-box modeling of flow in sewer systems with state-dependent diffusion”, Environmetrics, Vol.22, No.8, (2011), pp 946-961.

Stochastic Flow Forecasting



Constant state variance

Variance changing with state

see Breinholt A., Thordarson F.Ø., Møller J.K., Grum M., Mikkelsen P.S., Madsen H., “Grey-box modeling of flow in sewer systems with state-dependent diffusion”, Environmetrics, Vol.22, No.8, (2011), pp 946-961.

Stochastic Flow Forecasting

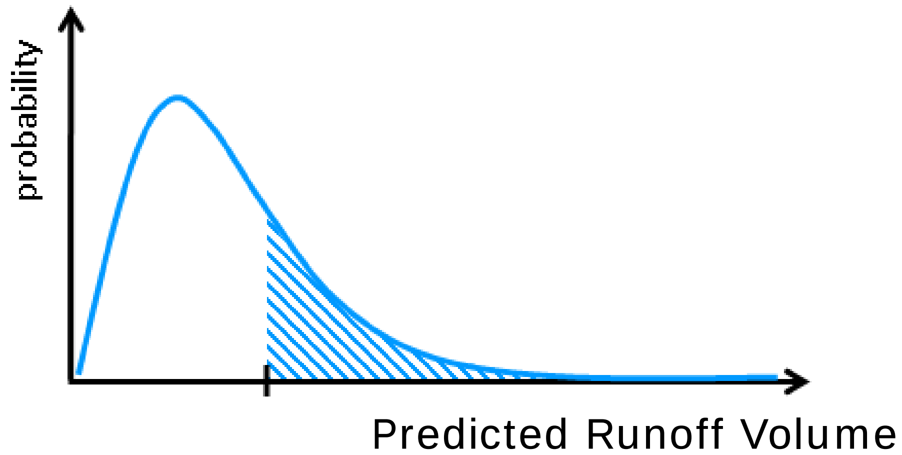
Why Greybox Modeling

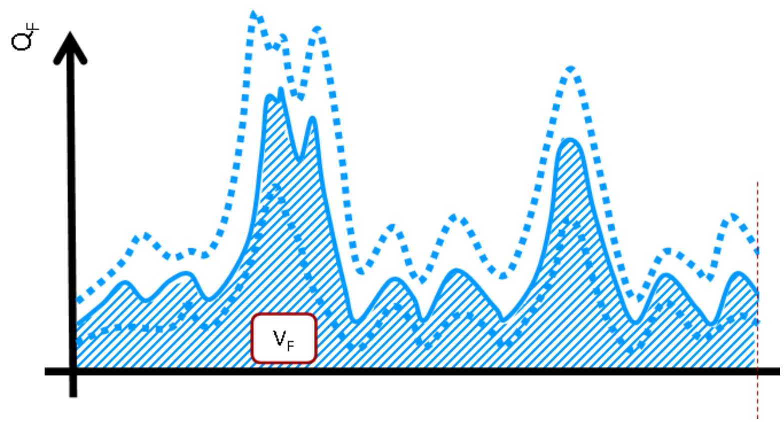
- Simple, fast models
- State-updating
- Auto-calibration
- Modeling and proper description of forecast uncertainties
- Allows to use statistical tools for model identification and verification (parameter tests, residual analysis)

Application

- Real-time control (Vezzaro et al.: A generalized Dynamic Overflow Risk Assessment (DORA) for urban drainage RTC – Session C1, Thursday 11:10)
- Software sensors

Stochastic Forecasts and Evaluation





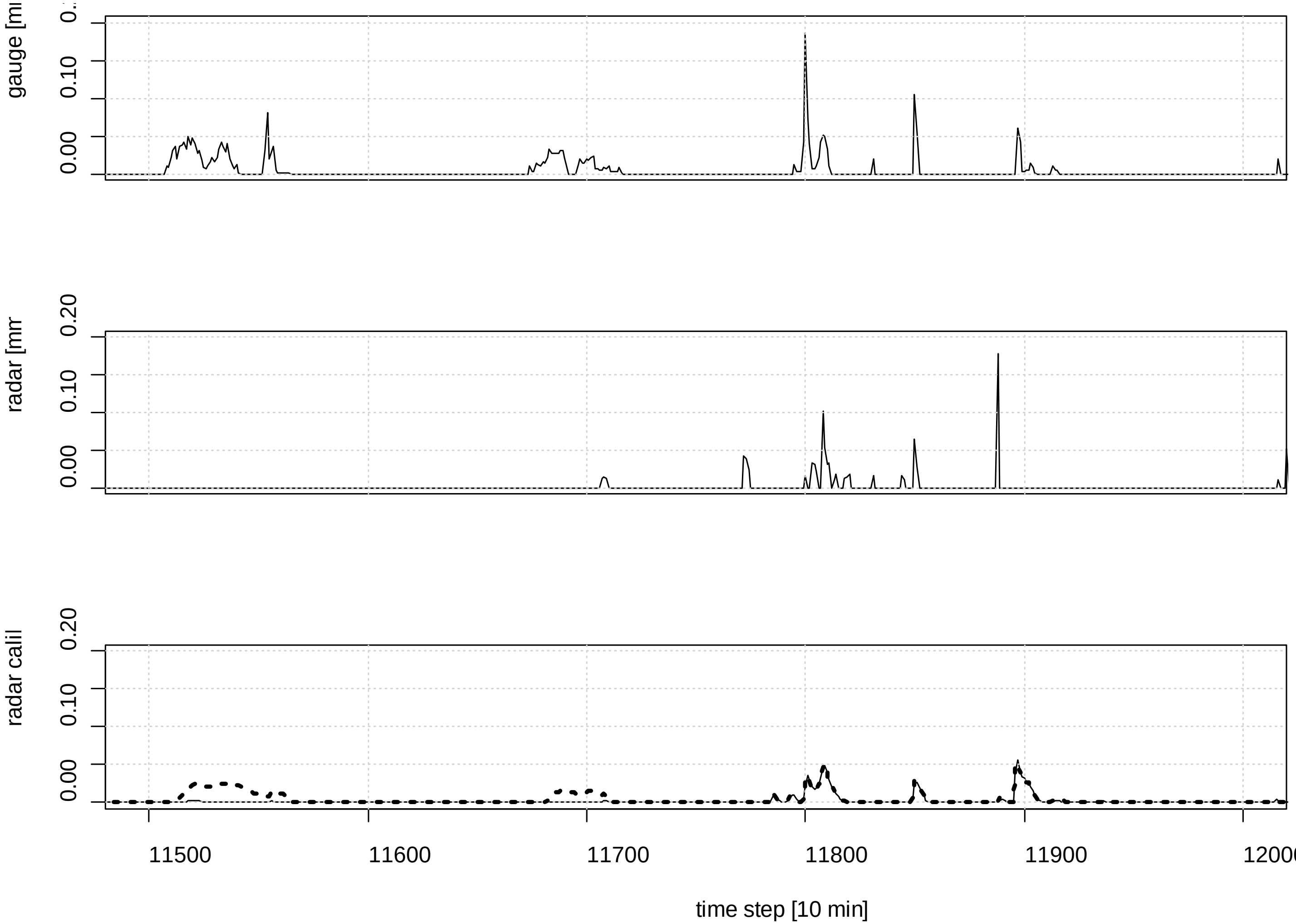
Evaluate 95% prediction interval

- Reliability (REL)
  (% observations outside pred. interval)
- Sharpness / Average Interval Length (ARIL)
  (width prediction interval)
- Skill Score (SK)
  (combines reliability and sharpness)

Advanced evaluation:

- CRPS

Results – Radar – Rain Gauge Merging: Error Marker



Results – Evaluating Runoff Forecasts

Evaluate 95% prediction intervals for 100min predictions of runoff volume

Model Input	Ballerup catchment			Damhusåen catchment		
	Rel	ARIL	Sk	Rel	ARIL	Sk
Rain gauge	5%	65%	1466	4%	116%	11777
Radar no adjustment	5%	56%	1378	6%	95%	12283
Radar Model 1	5%	56%	1342	6%	90%	10975
Radar Model 2	5%	64%	1403	6%	93%	11265
Radar Model 3	5%	59%	1339	5%	94%	10479

Discussion

- Merging of radar and raingauge data seems to improve runoff forecasts
- Issues in the considered approach:
  - Parameter estimation for radar–raingauge–merging should be based on flow observations
  - Applied simple Kalman filter for merging radar and raingauge data is not suitable for full scale implementation
    Alternative: e.g. Ensemble Kalman filter

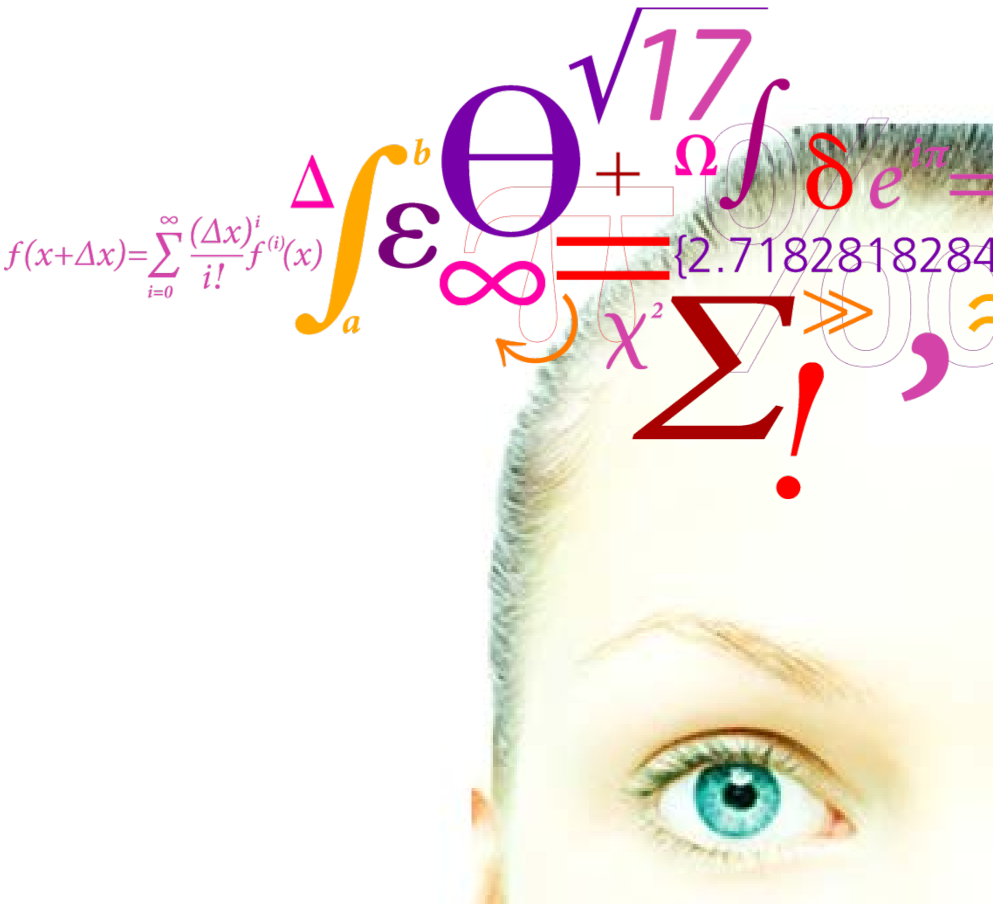
SWI
Storm- and Wastewater Informatics

swi.env.dtu.dk

”...close the knowledge gaps within prediction and control of current and future conditions in integrated urban wastewater systems”

- 2008-2013
- Budget 4 mio. €, half funded by Danish Council for Strategic Research, half by private companies
- Create components of an intelligent real-time decision support system

Thank you!



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Results – Radar – Rain Gauge Merging: Parameters

Model	a	σ_x	σ_R	σ_G
Model 1 (const. variances)	0.20	$1.23 \cdot 10^{-4}$	$8.12 \cdot 10^{-4}$	$9.12 \cdot 10^{-4}$
Model 2 (with correlation)	0.20	$1.51 \cdot 10^{-4}$	$8.12 \cdot 10^{-4}$	$3.80 \cdot 10^{-1}$
Model 3 (with error marker)	0.20	$1.23 \cdot 10^{-4}$	$8.12 \cdot 10^{-4}$	$9.12 \cdot 10^{-4}$