Computational Fluid Dynamics
Računska dinamika fluida

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NUMERIČKE METODE REŠAVANJA STRUJANJA VISKOZNOG NESTIŠLJIVOG FLUIDA

SA PRENOSOM TOPLOTE

2.1 OSNOVNE JEDNAČINE U MEHANICI FLUIDA

Navije-Stoksova jednačina

$$\rho\left(\frac{\partial v_i}{\partial t}+v_j v_{i,j}\right)=\sigma_{ij,j}+f_i^B$$

Konstitutivna relacija za napon

$$\sigma_{ij}=-p\delta_{ij}+2\mu e_{ij}$$

Tenzor brzine deformacije

$$e_{ij}=\frac{1}{2}\left(v_{i,j}+v_{j,i}\right)$$

Za nestišljivi fluid

$$\rho\left(\frac{\partial v_i}{\partial t}+v_j v_{i,j}\right)=-p_{,i}+\mu v_{i,jj}+f_i^B$$

Jednačina kontinuiteta

$$\frac{D\rho}{Dt}+\rho\frac{\partial v_i}{\partial x_i}=0$$

Za nestišljivi fluid

$$v_{i,i}=0$$

Jednačina provođenja toplote

$$\rho c_p\left(\frac{\partial \theta}{\partial t}+v_i\theta_{,i}\right)=\left(k\theta_i\right)_{,i}+q^B$$

2.2 IMPLICITNE METODE REŠAVANJA STRUJANJA LAMINARNOG VISKOZNOG NESTIŠLJIVOG FLUIDA SA PRENOSOM TOPLOTE

2.2.1 Mešovita (brzine-pritisci) v-p formulacija

$$\rho\left(\frac{\partial v_i}{\partial t}+v_j v_{i,j}\right)=-p_{,i}+\mu v_{i,jj}+f_i^B \quad \textbf{(2.2.8)} \qquad v_{i,i}=0 \quad \textbf{(2.2.2)}$$

Primena Galerkinovog postupka

$$\rho\int_V H_\alpha \frac{\partial v_i}{\partial t}dV+\rho\int_V H_\alpha v_j v_{i,j}dV=-\int_V H_\alpha p_{,i}dV+\int_V \mu H_\alpha v_{i,jj}dV+\int_V H_\alpha f_i^B dV \quad \textbf{(2.2.9)}$$

$$\int_V G_\delta v_{i,i}dV=0 \quad \textbf{(2.2.10)}$$

Parcijalna integracija i prevođenje zapreminskog u površinski integral

$$\rho\int_V H_\alpha \frac{\partial v_i}{\partial t}dV+\rho\int_V H_\alpha v_j v_{i,j}dV-\int_V H_{\alpha,i}\,p\,dV+\int_V \mu H_{\alpha,j}v_{i,j}dV=\int_V H_\alpha f_i^B dV+\int_S H_\alpha\left(-pn_i+\mu v_{i,j}n_j\right)dS \quad \textbf{(2.2.11)}$$

Korišćenjem interpolacijskih funkcija za brzinu i pritisak fluida $v_i = H_\alpha v_{i\alpha} \quad p = G_\delta p_\delta$ **(2.2.11)**

Prikaz tipa elementa i broja nepoznatih veličina po elementu

Tip elementa	Broj ~vorova po elementu	Broj nepoznatih veli~ina po elementu	
		Brzina	Pritisaka
2-D	4	4	1
	9	9	4
3-D	8	8	1
	21	21	8
	27	27	8

Matrični oblik jednačina

$$\begin{bmatrix} \mathbf{M_v} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{v}} \\ \dot{\mathbf{p}} \end{Bmatrix} + \begin{bmatrix} \mathbf{K_{vv}} + \mathbf{K}_{\mu\mathbf{v}} & \mathbf{K_{vp}} \\ \mathbf{K_{vp}^T} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{v} \\ \mathbf{p} \end{Bmatrix} = \begin{Bmatrix} \mathbf{R_B} + \mathbf{R_S} \\ \mathbf{0} \end{Bmatrix} \quad \textbf{(2.2.15)}$$

Brzina i pritisak na kraju koraka

$${}^{t+\Delta t}v_{i\alpha} = {}^{t+\Delta t}v_{i\alpha}^{(m-1)} + \Delta v_{i\alpha}^{(m)} \qquad {}^{t+\Delta t}p_\delta = {}^{t+\Delta t}p_\delta^{(m-1)} + \Delta p_\delta^{(m)}$$

Inkrementalno-iterativne jednačine

$$\left[\rho\frac{1}{\Delta t}\int_V H_\alpha H_\beta dV\right]\left(\Delta v_{\beta i}^{(m)}\right)+\left[\rho\int_V H_\alpha H_\gamma\,{}^{t+\Delta t}v_{\gamma j}^{(m-1)}H_{\beta,j}dV\right]\left(\Delta v_{\beta i}^{(m)}\right)+\left[\rho\int_V H_\alpha H_{\gamma,j}\,{}^{t+\Delta t}v_{\gamma i}^{(m-1)}H_\beta dV\right]\left(\Delta v_{\beta j}^{(m)}\right)$$

$$\left[\int_V \mu H_{\alpha,j}H_{\beta,j}dV\right]\left(\Delta v_{\beta i}^{(m)}\right)-\left[\int_V H_{\alpha,i}G_\delta dV\right]\left(\Delta p_\delta^{(m)}\right)=\int_V H_\alpha f_i^B dV+\int_S H_\alpha\left(-pn_i+v_{i,j}n_j\right)dS-$$

$$\left[\rho\frac{1}{\Delta t}\int_V H_\alpha H_\beta dV\right]\left({}^{t+\Delta t}v_{\beta i}^{(m-1)}-{}^{t}v_{\beta i}\right)-\left[\rho\int_V H_\alpha H_\gamma\,{}^{t+\Delta t}v_{\gamma j}^{(m-1)}H_{\beta,j}dV\right]\left({}^{t+\Delta t}v_{\beta i}^{(m-1)}\right)-\left[\int_V \mu H_{\alpha,j}H_{\beta,j}dV\right]\left({}^{t+\Delta t}v_{\beta i}^{(m-1)}\right)-$$

$$\left[\int_V H_{\alpha,i}G_\delta dV\right]\left({}^{t+\Delta t}p_\delta^{(m-1)}\right) \quad \textbf{(2.2.26)}$$

$$\left[\int_V G_\delta H_{\alpha,i}dV\right]\left(\Delta v_{\alpha i}^{(m)}\right)=-\left[\int_V G_\delta H_{\alpha,i}dV\right]\left({}^{t+\Delta t}v_{\alpha i}^{(m-1)}\right) \quad \textbf{(2.2.27)}$$

Matrični oblik jednačina

$$\begin{bmatrix}\frac{1}{\Delta t}\mathbf{M_v}+{}^{t+\Delta t}\mathbf{K}_{\mathbf{vv}}^{(m-1)}+{}^{t+\Delta t}\mathbf{K}_{\mu\mathbf{v}}^{(m-1)}+{}^{t+\Delta t}\mathbf{J}_{\mathbf{vv}}^{(m-1)} & \mathbf{K_{vp}}\\ \mathbf{K_{vp}^T} & \mathbf{0}\end{bmatrix}\begin{Bmatrix}\Delta\mathbf{v}^{(m)}\\ \Delta\mathbf{p}^{(m)}\end{Bmatrix}=\begin{Bmatrix}{}^{t+\Delta t}\mathbf{F}_{\mathbf{v}}^{(m-1)}\\ {}^{t+\Delta t}\mathbf{F}_{\mathbf{p}}^{(m-1)}\end{Bmatrix} \quad \textbf{(2.2.28)}$$

2.2.2 PENALTI formulacija za strujanje fluida

Uslov nestišljivosti $$v_{i,i} + \frac{p}{\lambda} = 0 \quad \textbf{(2.2.38)}$$

Navije-Stoksova jednačina $$\rho\left(\frac{\partial v_i}{\partial t} + v_j v_{i,j}\right) = \lambda v_{j,ij} + \mu\left(v_{i,j} + v_{j,i}\right)_{,j} + f_i^B \quad \textbf{(2.2.40)}$$

Matrični oblik jednačine

$$\left(\frac{1}{\Delta t}\mathbf{M}_{\mathbf{v}} + {}^{t+\Delta t}\mathbf{K}_{\mathbf{vv}}^{(m-1)} + {}^{t+\Delta t}\mathbf{K}_{\mu\mathbf{v}}^{(m-1)} + {}^{t+\Delta t}\hat{\mathbf{K}}_{\mu\mathbf{v}}^{(m-1)} + {}^{t+\Delta t}\mathbf{J}_{\mathbf{vv}}^{(m-1)} + \mathbf{K}_{\lambda\mathbf{v}}\right)\Delta\mathbf{v}^{(m)} = {}^{t+\Delta t}\hat{\mathbf{F}}_{\mathbf{v}}^{(m-1)} \quad \textbf{(2.2.41)}$$

2.2.3 Mešovita (brzine-pritisci-temperature, v-p-θ) formulacija

$$\begin{bmatrix} \frac{1}{\Delta t}\mathbf{M}_{\mathbf{v}} + {}^{t+\Delta t}\mathbf{K}_{\mathbf{vv}}^{(m-1)} + {}^{t+\Delta t}\mathbf{K}_{\mu\mathbf{v}}^{(m-1)} + {}^{t+\Delta t}\mathbf{J}_{\mathbf{vv}}^{(m-1)} & \mathbf{K}_{\mathbf{vp}} & \mathbf{0} \\ \mathbf{K}_{\mathbf{vp}}^{\mathbf{T}} & \mathbf{0} & \mathbf{0} \\ {}^{t+\Delta t}\mathbf{K}_{\theta v}^{(m-1)} & \mathbf{0} & \frac{1}{\Delta t}\mathbf{M}_{\theta} + {}^{t+\Delta t}\mathbf{K}_{\theta\theta}^{(m-1)} + {}^{t+\Delta t}\mathbf{J}_{\theta\theta}^{(m-1)} \end{bmatrix} \begin{Bmatrix} \Delta\mathbf{v}^{(m)} \\ \Delta\mathbf{p}^{(m)} \\ \Delta\theta^{(m)} \end{Bmatrix} = \begin{Bmatrix} {}^{t+\Delta t}\mathbf{F}_{\mathbf{v}}^{(m-1)} \\ {}^{t+\Delta t}\mathbf{F}_{\mathbf{p}}^{(m-1)} \\ {}^{t+\Delta t}\mathbf{F}_{\theta}^{(m-1)} \end{Bmatrix}$$

(2.2.64)

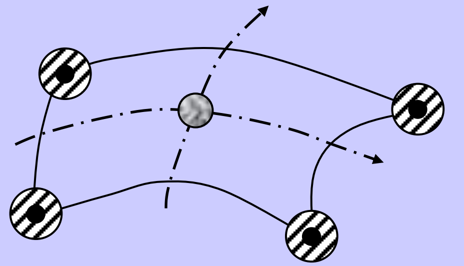
2.2.4 PENALTI formulacija za strujanje fluida sa prenosom toplote

$$\begin{bmatrix} \frac{1}{\Delta t}\mathbf{M}_{\mathbf{v}} + {}^{t+\Delta t}\mathbf{K}_{\mathbf{vv}}^{(m-1)} + {}^{t+\Delta t}\mathbf{K}_{\mu\mathbf{v}}^{(m-1)} + \\ {}^{t+\Delta t}\hat{\mathbf{K}}_{\mu\mathbf{v}}^{(m-1)} + {}^{t+\Delta t}\mathbf{J}_{\mathbf{vv}}^{(m-1)} + \mathbf{K}_{\lambda\mathbf{v}} & \mathbf{0} \\ {}^{t+\Delta t}\mathbf{K}_{\theta v}^{(m-1)} & \frac{1}{\Delta t}\mathbf{M}_{\theta} + {}^{t+\Delta t}\mathbf{K}_{\theta\theta}^{(m-1)} + {}^{t+\Delta t}\mathbf{J}_{\theta\theta}^{(m-1)} \end{bmatrix} \begin{Bmatrix} \Delta\mathbf{v}^{(m)} \\ \Delta\theta^{(m)} \end{Bmatrix} = \begin{Bmatrix} {}^{t+\Delta t}\hat{\mathbf{F}}_{\mathbf{v}}^{(m-1)} \\ {}^{t+\Delta t}\mathbf{F}_{\theta}^{(m-1)} \end{Bmatrix} \qquad (2.2.65)$$

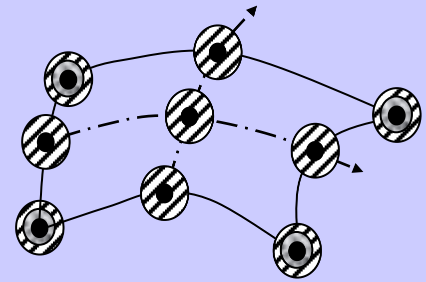
Tipovi elemenata za 2D i 3D analizu

Brzina fluida
Pritiak fluida
Temper. fluida

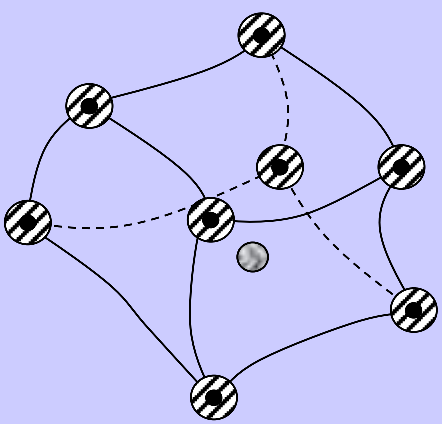
2D/4, v-4, p-1, θ-4



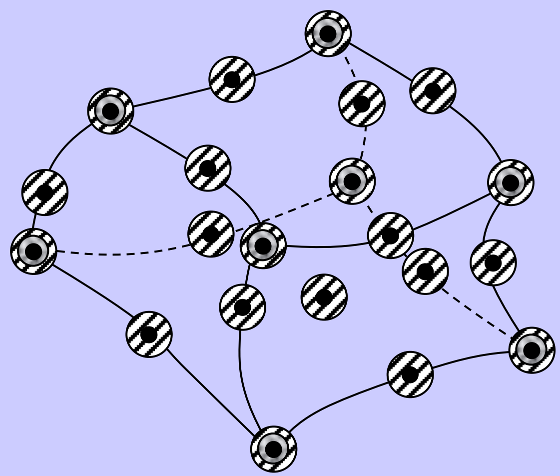
2D/9, v-9, p-4, θ-9



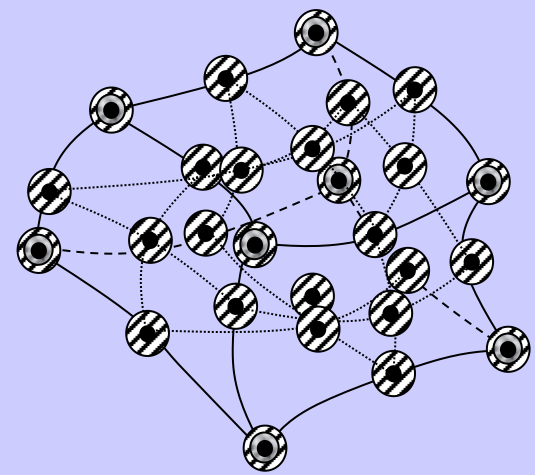
3D/8, v-8, p-1, θ-8



3D/21, v-21, p-8, θ-21



3D/27, v-27, p-8, θ-27



2.3 EKSPLICITNA METODA IZ DVA KORAKA

Jednačina kontinuiteta $\frac{\partial \rho}{\partial t} + (\rho v_i)_{,i} = 0$ (2.3.1)

Navije-Stoksova jednačina $\rho\left(\frac{\partial v_i}{\partial t} + v_j v_{i,j}\right) + p_{,i} - \mu\left(v_{i,j} + v_{j,i}\right)_{,j} - \rho f_i^B = 0$ (2.3.2)

Jednačina stanja $p = p(\rho)$ (2.3.3)

Brzina zvuka $c^2 = \frac{\partial p}{\partial \rho}$ (2.3.4)

Jednačina kontinuiteta $\frac{\partial p}{\partial t} + v_i p_{,i} + \rho c^2 v_{i,i} = 0$ (2.3.6)

Navije-Stoksova jednačina u matričnom obliku $\mathbf{M_v \dot{v}} + \mathbf{K_{vv} v} + \mathbf{K_{vp} p} - \mathbf{F_B} - \mathbf{F_S} = \mathbf{0}$ (2.3.8)

Jednačina kontinuiteta u matričnom obliku $\mathbf{M_p \dot{p}} + \mathbf{K_{pv} v} + \mathbf{K_{pp} p} = \mathbf{0}$ (2.3.9)

Selektivna 'lumping' dvo-kora~na eksplicitna šema

Prvi korak

$$\overline{\mathbf{M}}_\mathbf{v} \mathbf{v}^{n+1/2} = \overline{\mathbf{M}}_\mathbf{v} \mathbf{v}^n - \frac{\Delta t}{2}\left(\mathbf{K}_\mathbf{vv}^n \mathbf{v}^n + \mathbf{K}_\mathbf{vp} \mathbf{p}^n - \mathbf{F}_\mathbf{B}^n - \mathbf{F}_\mathbf{S}^n\right) \quad (2.3.11)$$

$$\overline{\mathbf{M}}_\mathbf{p} \mathbf{p}^{n+1/2} = \tilde{\mathbf{M}}_\mathbf{p} \mathbf{p}^n - \frac{\Delta t}{2}\left(\mathbf{K}_\mathbf{pv} \mathbf{v}^n + \mathbf{K}_\mathbf{pp}^n \mathbf{p}^n\right) = \mathbf{0} \quad (2.3.12)$$

Drugi korak

$$\overline{\mathbf{M}}_\mathbf{v} \mathbf{v}^{n+1} = \overline{\mathbf{M}}_\mathbf{v} \mathbf{v}^n - \Delta t\left(\mathbf{K}_\mathbf{vv}^{n+1/2} \mathbf{v}^{n+1/2} + \mathbf{K}_\mathbf{vp} \mathbf{p}^{n+1/2} - \mathbf{F}_\mathbf{B}^n - \mathbf{F}_\mathbf{S}^{n+1/2}\right) \quad (2.3.13)$$

$$\overline{\mathbf{M}}_\mathbf{p} \mathbf{p}^{n+1} = \tilde{\mathbf{M}}_\mathbf{p} \mathbf{p}^n - \Delta t\left(\mathbf{K}_\mathbf{pv} \mathbf{v}^{n+1/2} + \mathbf{K}_\mathbf{pp}^{n+1/2} \mathbf{p}^{n+1/2}\right) = \mathbf{0} \quad (2.3.14)$$

Selektivni 'lumping' parametar e $\tilde{\mathbf{M}}_\mathbf{p} = e\overline{\mathbf{M}}_\mathbf{p} + (1-e)\mathbf{M}_\mathbf{p}$ (2.3.15)

2.4 EKSPLICITNO-IMPLICITNA TRO-STEPENA METODA ZA REŠAVANJE STRUJANJA FLUIDA

Trostepena šema

$$\frac{v_i^{n+1/3}-v_i^n}{\Delta t/3}=-v_j^n v_{i,j}^n-\frac{p_{,i}^n}{\rho}+\nu\left(v_{i,j}^n+v_{j,i}^n\right)_{,j}+f_i^{\,n} \quad (2.4.3)$$

$$\frac{v_i^{n+1/2}-v_i^n}{\Delta t/2}=-v_j^{n+1/3} v_{i,j}^{n+1/3}-\frac{p_{,i}^n}{\rho}+\nu\left(v_{i,j}^{n+1/3}+v_{j,i}^{n+1/3}\right)_{,j}+f_i^{\,n+1/3} \quad (2.4.4)$$

$$\frac{v_i^{n+1}-v_i^n}{\Delta t}=-v_j^{n+1/2} v_{i,j}^{n+1/2}-\frac{p_{,i}^{n+1}}{\rho}+\nu\left(v_{i,j}^{n+1/2}+v_{j,i}^{n+1/2}\right)_{,j}+f_i^{\,n+1/2} \quad (2.4.5)$$

Primena Galerkina

$$\frac{1}{\rho}\int_V H_{\alpha,i}\, p_{,i}^{n+!} dV=-\frac{1}{\Delta t}\int_V H_\alpha v_{i,i}^n dV-\int_V H_{\alpha,i} v_j^{n+1/2} v_{i,j}^{n+1/2} dV+\int_V H_{\alpha,i} f_i^{\,n+1/2} dV-\int_S H_\alpha\left(\frac{v_i^{n+1}-v_i^n}{\Delta t}\right) n_i dS \quad (2.4.10)$$

Algoritam rešavanja

$$\boxed{\{\overline{\mathbf{M}}\}\{\mathbf{v}^{n+1/3}\}=\left\{\{\overline{\mathbf{M}}\}\{\mathbf{v}^n\}+\frac{\Delta t}{3}\left(\mathbf{F}_\mathbf{p}^n+\mathbf{F}_\nu^n+\mathbf{F}_\mathbf{B}^n+\mathbf{F}_\mathbf{S}^n\right)\right\}} \quad (2.4.13)$$

$$\Downarrow$$

$$\boxed{\{\overline{\mathbf{M}}\}\{\mathbf{v}^{n+1/2}\}=\left\{\{\overline{\mathbf{M}}\}\{\mathbf{v}^n\}+\frac{\Delta t}{2}\left(\mathbf{F}_\mathbf{p}^n+\mathbf{F}_\nu^{n+1/3}+\mathbf{F}_\mathbf{B}^{n+1/3}+\mathbf{F}_\mathbf{S}^{n+1/3}\right)\right\}} \quad (2.4.13)$$

$$\Downarrow$$

$$\boxed{[\mathbf{K}_\mathbf{pp}]\{\mathbf{p}^{n+1}\}=\{\mathbf{F}_\mathbf{p}^n+\mathbf{F}_\mathbf{p}^{n+!/2}\}} \quad (2.4.11)$$

$$\Downarrow$$

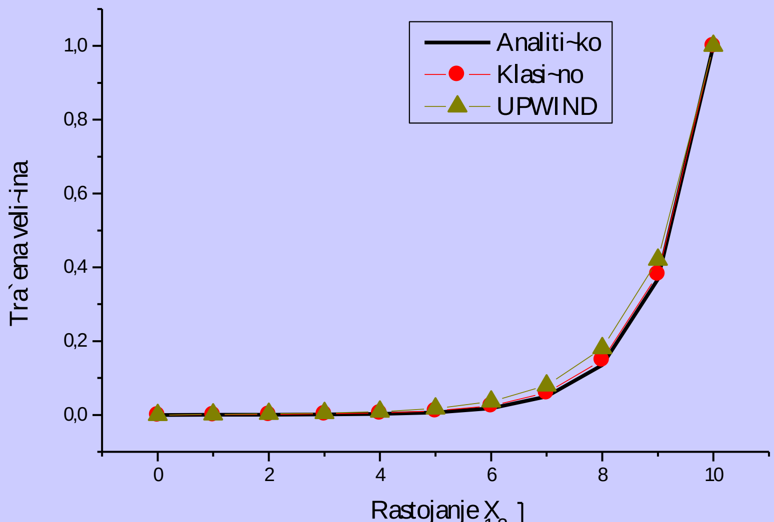
$$\boxed{\{\overline{\mathbf{M}}\}\{\mathbf{v}^{n+1}\}=\left\{\{\overline{\mathbf{M}}\}\{\mathbf{v}^n\}+\frac{\Delta t}{3}\left(\mathbf{F}_\mathbf{p}^{n+!}+\mathbf{F}_\nu^{n+1/2}+\mathbf{F}_\mathbf{B}^{n+1/2}+\mathbf{F}_\mathbf{S}^{n+1/2}\right)\right\}} \quad (2.4.16)$$

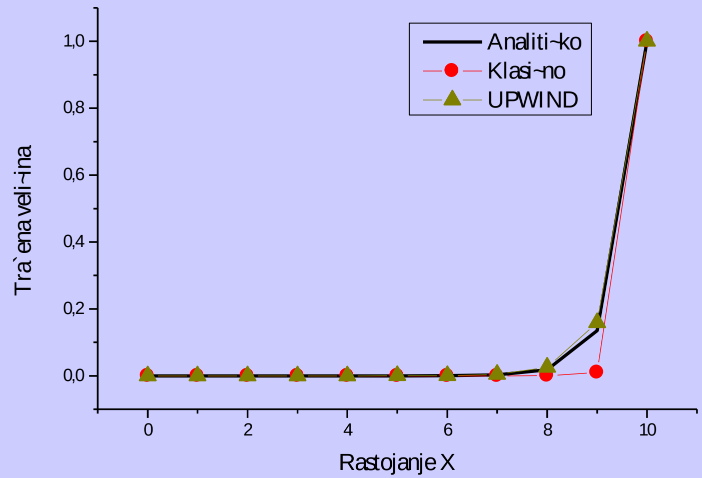
2.5 PRIMENA "UPWIND" TEHNIKE NA KONVEKTIVNO DOMINANTNA STRUJANJA FLUIDA

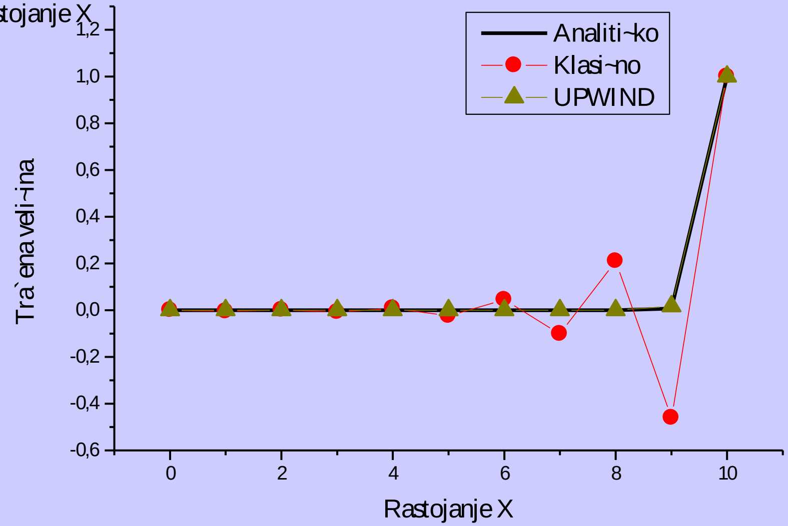
Jednodimenziona advektivno-difuzna jednačina

$$u\frac{\partial \varphi}{\partial x} = k\frac{\partial^2 \varphi}{\partial x^2} \qquad (2.5.1)$$

Pekleov broj mreže $\alpha = uh/(2k)$





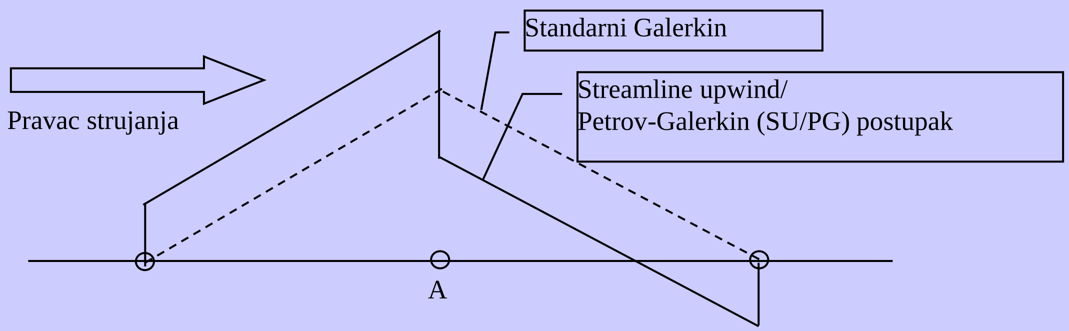


2.5.3 Streamline UPWIND Petrov-Galerkin metoda

Tenzor veštačke diuzivnosti $\bar{k}_{ij} = \bar{k}\hat{u}_i\hat{u}_j$ **(2.5.12)** **gde je** $\hat{u}_i = \dfrac{u_i}{\|\mathbf{u}\|}$

Korigovane interpolacijske funkcije $\bar{h} = h + \bar{k}\hat{u}_j h_{,j} \dfrac{1}{\|\mathbf{u}\|}$ **(2.5.18)**

Grafička interpretacija



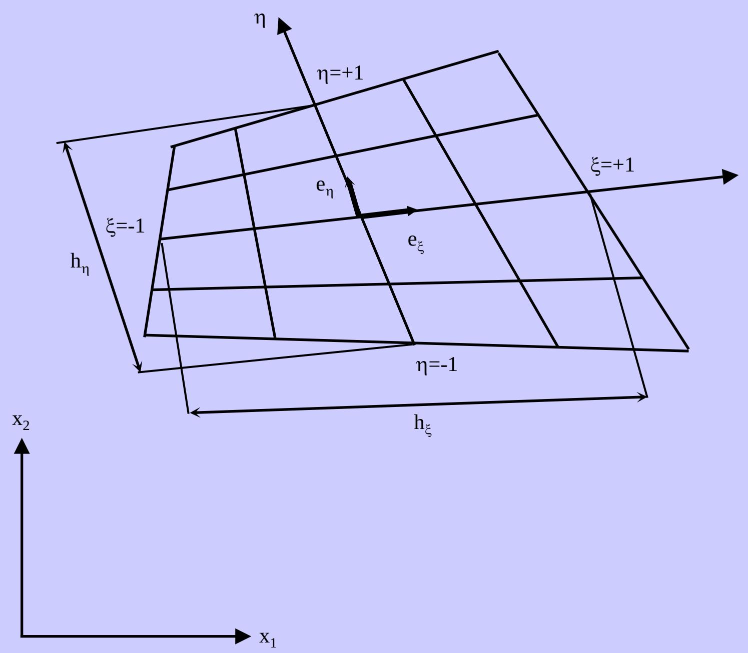
1D

$$\bar{k} = \left(\xi u_\xi h_\xi + \eta u_\eta h_\eta\right)/2 \quad \textbf{(2.5.19)}$$

$$\xi = \left(\coth \alpha_\xi\right) - 1/\alpha_\xi \qquad \eta = \left(\coth \alpha_\eta\right) - 1/\alpha_\eta$$

$$\alpha_\xi = u_\xi h_\xi /(2k) \quad \alpha_\eta = u_\eta h_\eta /(2k) \qquad \textbf{(2.5.20)}$$

$$u_\xi = \mathbf{e}_\xi \cdot \mathbf{u} \qquad u_\eta = \mathbf{e}_\eta \cdot \mathbf{u}$$



2D

2.6 TEJLOR-GALERKINOVA METODA
ZA NESTACIONARNE KONVEKTIVNO-DIFUZNE PROBLEME

Burgerova viskozna 1D jednačina

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right)=\mu\frac{\partial^2 u}{\partial x^2} \qquad \textbf{(2.6.1)}$$

Tejlorov red

$$u^{n+1}=u^n+\frac{\partial u^n}{\partial t}\Delta t+\frac{1}{2!}\frac{\partial^2 u^n}{\partial t^2}(\Delta t)^2+O(\Delta t)^3 \qquad \textbf{(2.6.2)}$$

Inkrementalna jedna~ina sa stabilizacionim delom

$$\frac{u^{n+1}-u^n}{\Delta t}=\left(-u^n\frac{\partial u^n}{\partial x}+\frac{\mu}{\rho}\frac{\partial^2 u^n}{\partial x^2}\right)+\frac{\Delta t}{2}\left[-u^n\frac{\partial}{\partial x}\left(-u^n\frac{\partial u^n}{\partial x}+\frac{\mu}{\rho}\frac{\partial^2 u^n}{\partial x^2}\right)+\frac{\mu}{\rho}\frac{\partial^2}{\partial x^2}\left(\frac{u^{n+1}-u^n}{\Delta t}\right)\right] \qquad \textbf{(2.6.7)}$$

2.7 STABILNOST DVOSTEPENE LAX-WENDROFF I TROSTEPENE METODE

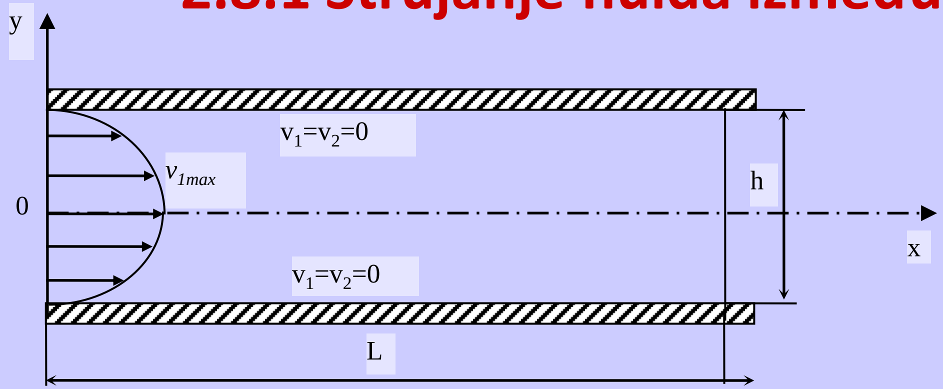
Dvostepena Lax-Wendrof-ova metoda

$$f(t+\Delta t/2)=f(t)+\frac{\Delta t}{2}\frac{\partial f(t)}{\partial t}$$
$$f(t+\Delta t)=f(t)+\Delta t\frac{\partial f(t+\Delta t/2)}{\partial t} \qquad \textbf{(2.7.4)}$$

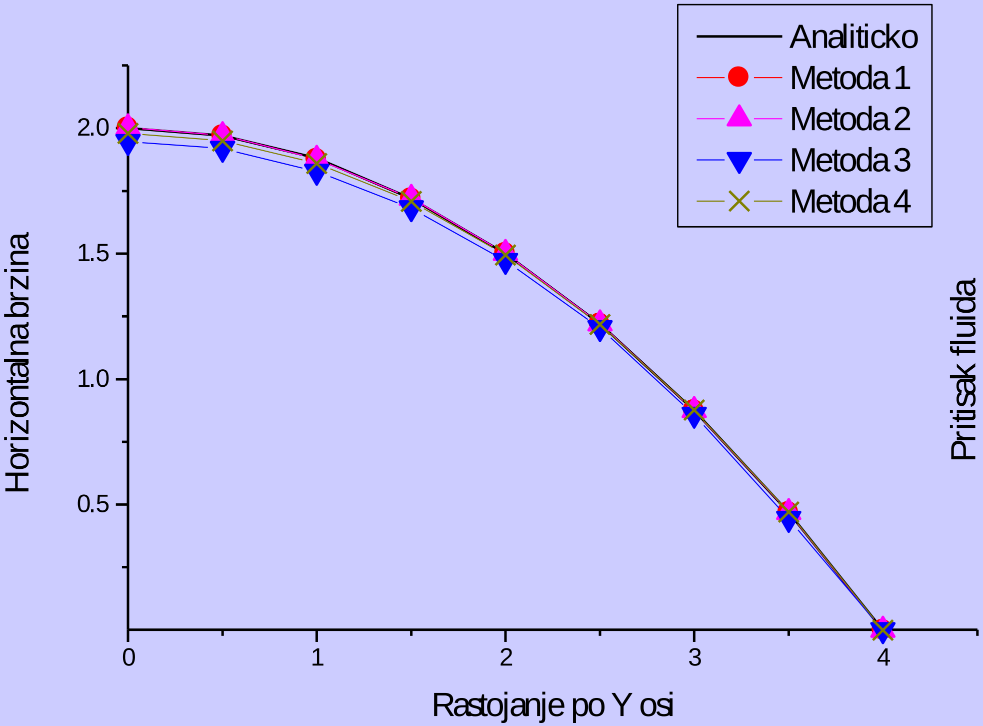
Trostepena stabilizaciona metoda

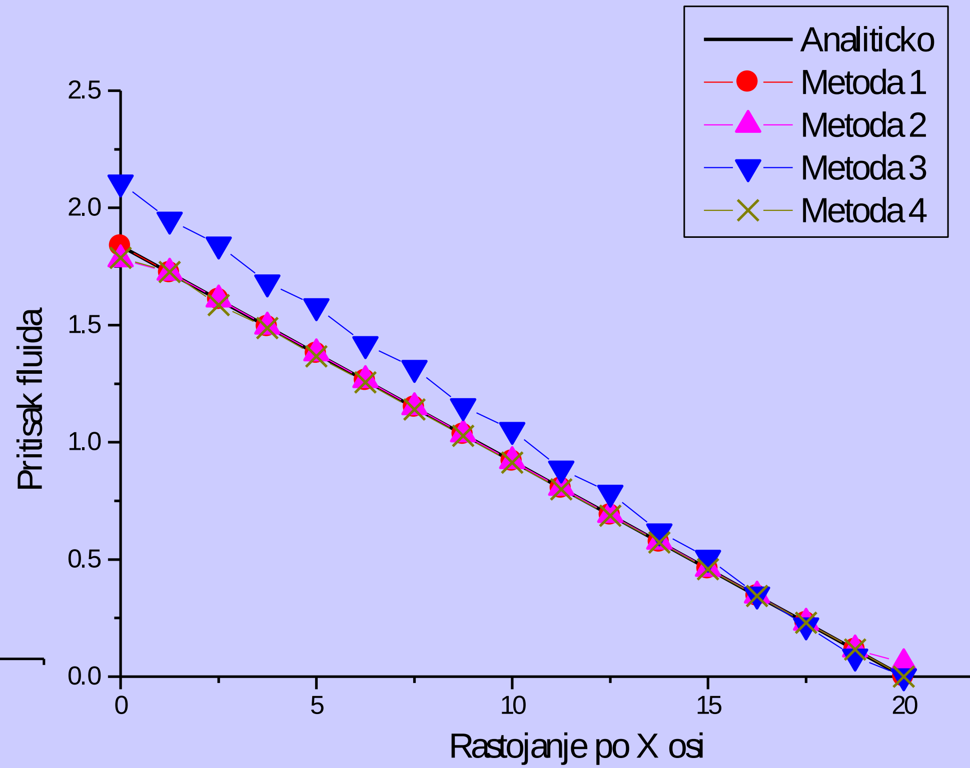
$$f(t+\Delta t/3)=f(t)+\frac{\Delta t}{3}\frac{\partial f(t)}{\partial t}$$
$$f(t+\Delta t/2)=f(t)+\frac{\Delta t}{2}\frac{\partial f(t+\Delta t/3)}{\partial t} \qquad \textbf{(2.7.5)}$$
$$f(t+\Delta t)=f(t)+\Delta t\frac{\partial f(t+\Delta t/2)}{\partial t}$$

2.8.1 Strujanje fluida između dve nepokretne ploče



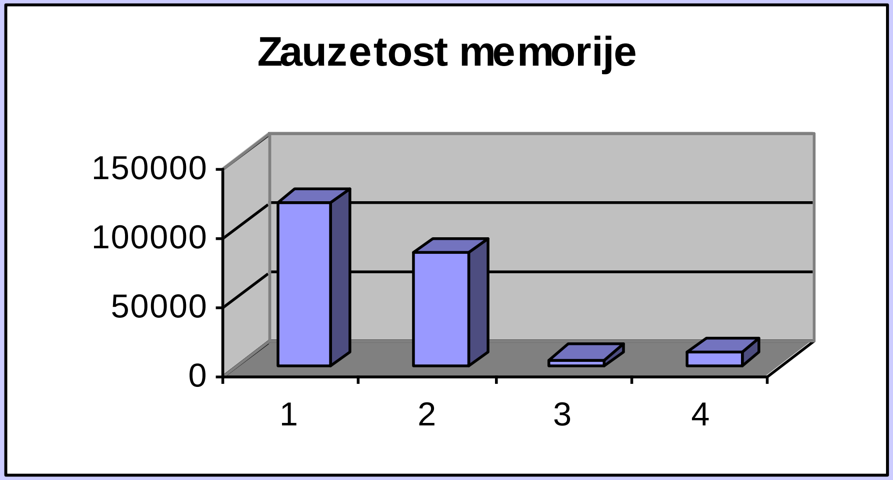
L=20 m
h=8 m
μ=0.3675 kg/m/s
v_{1max}=2.0 m/s

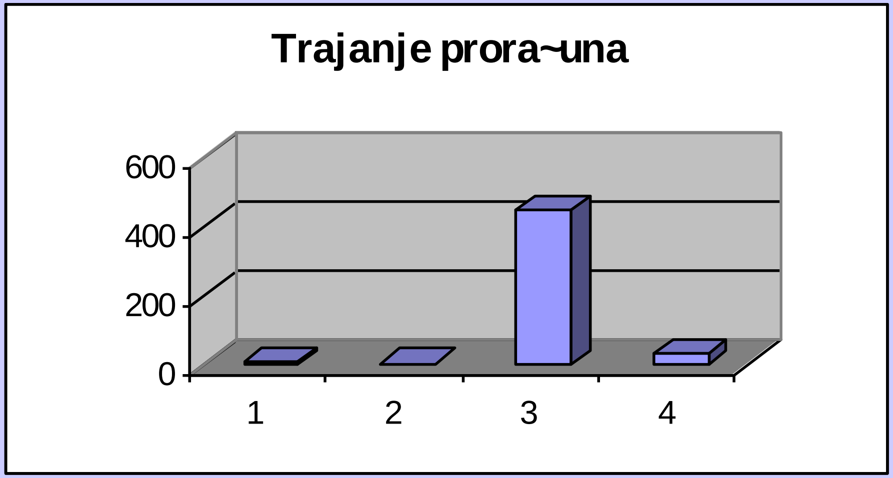




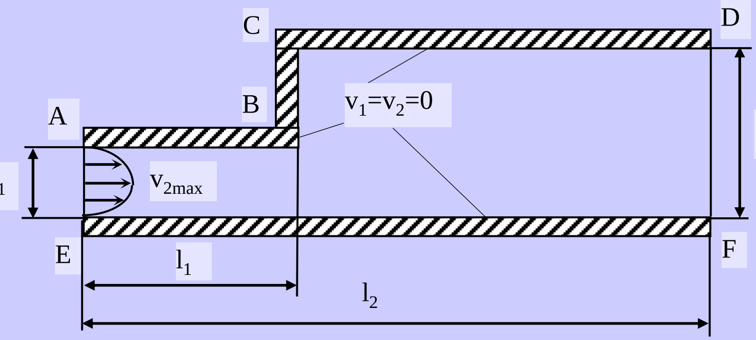
Uporedni prikaz zauzetosti memorije i trajanja proračuna

Metoda 1 : Mešovita formulacija (9/4 element), implicitna metoda
Metoda 2 : Penalti formulacija (4/1 element), implicitna metoda
Metoda 3 : Čisto eksplicitna metoda (4/4 element)
Metoda 4 : Eksplicitno-implicitna metoda (4/4 element)

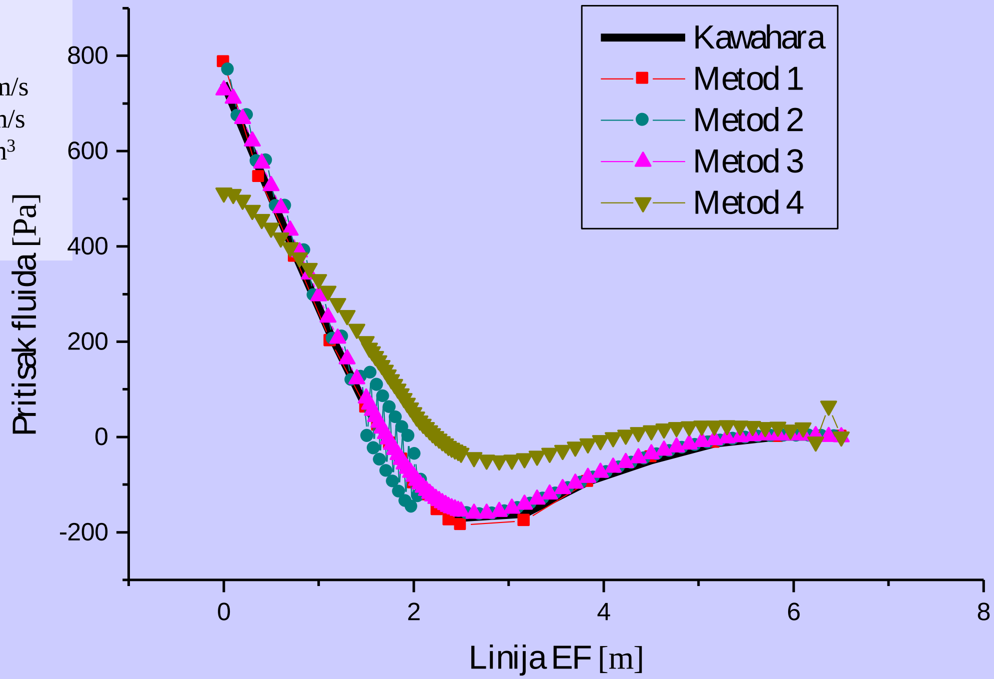




2.8.2 Stacionarno ravansko strujanje fluida kroz kanal sa proširenjem

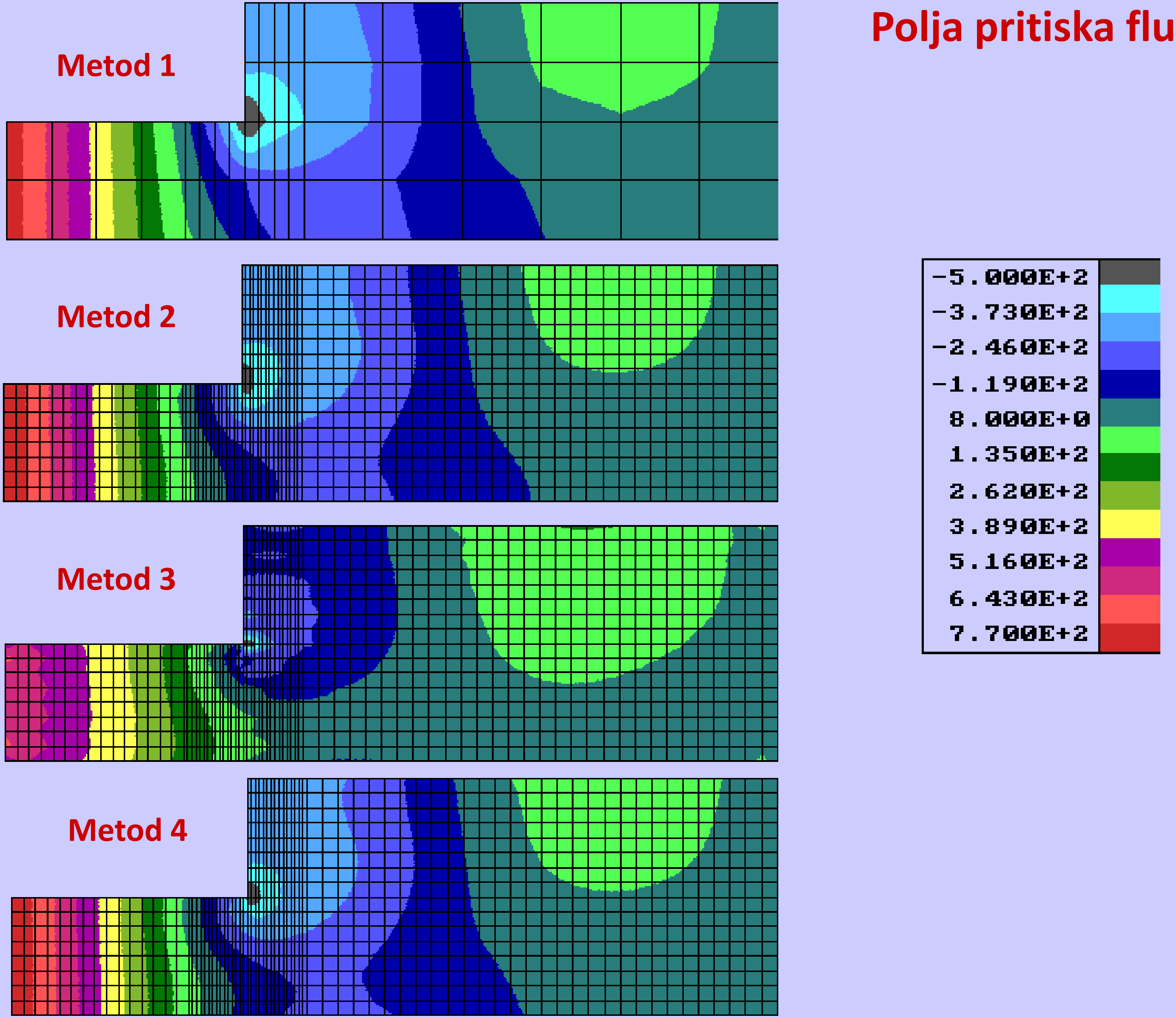


$d_1=1$ m
$d_2=2$ m
$l_1=2$ m
$l_2=6.5$ m
$v_{2max}=2$ m/s
$\mu=1$ kg/m/s
$\rho=1$ kg/m^3
$R_e=60$

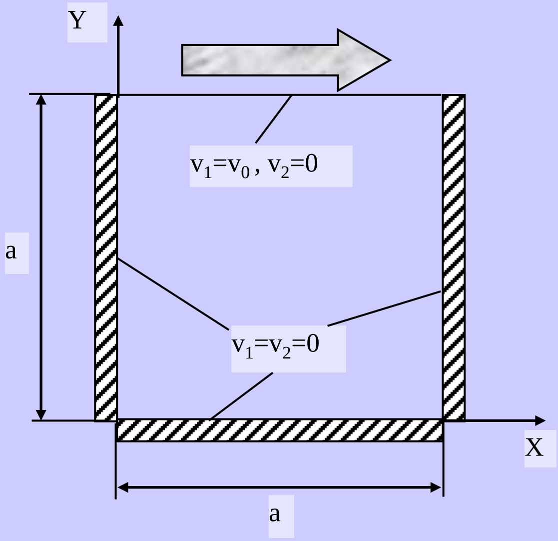


Legenda	Broj koraka	Zauzetost memorije	Trajanje prora~una
Metoda1	1	2.4Mba	40 sekundi
Metoda2	1	1.05Mba	5 sekundi
Metoda3	7000, $\Delta t= 4 x10^{-5}$	0.04Mba	7654 sekundi
Metoda4	500, $\Delta t= 6 x10^{-4}$	0.5Mba	1000 sekudni

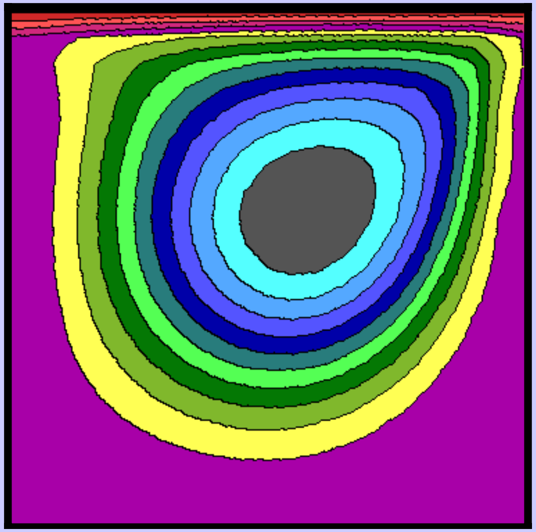
Polja pritiska fluida



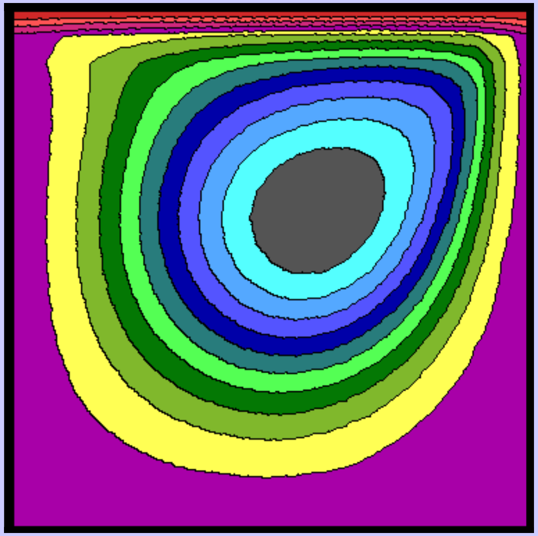
2.8.3 Strujanje fluida u šupljini pri zadatoj brzini na jednoj stranici



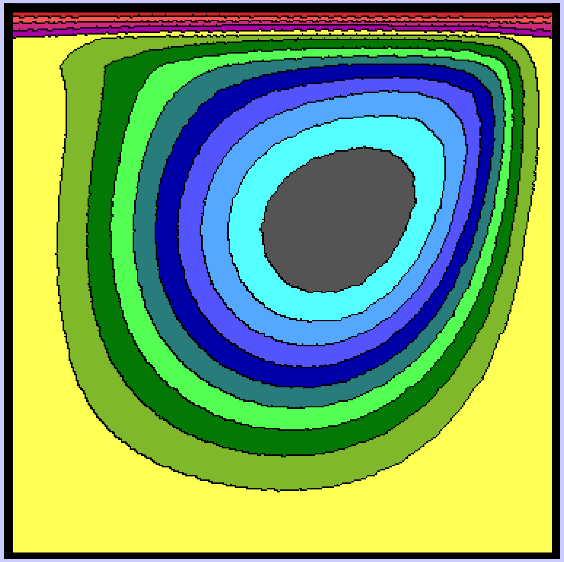
Metoda 1



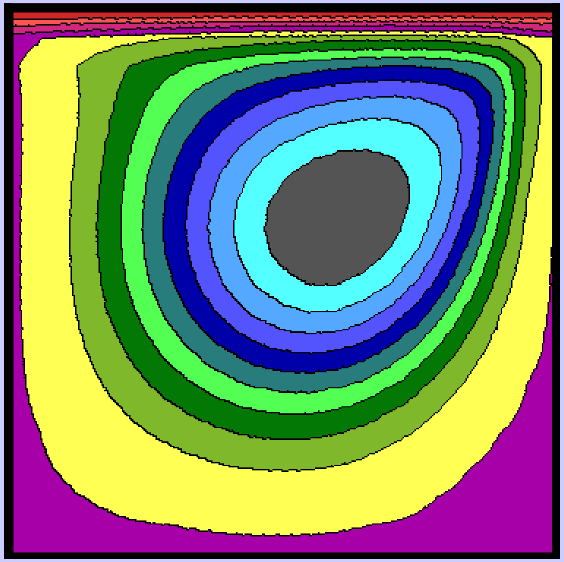
Metoda 2



Metoda 3

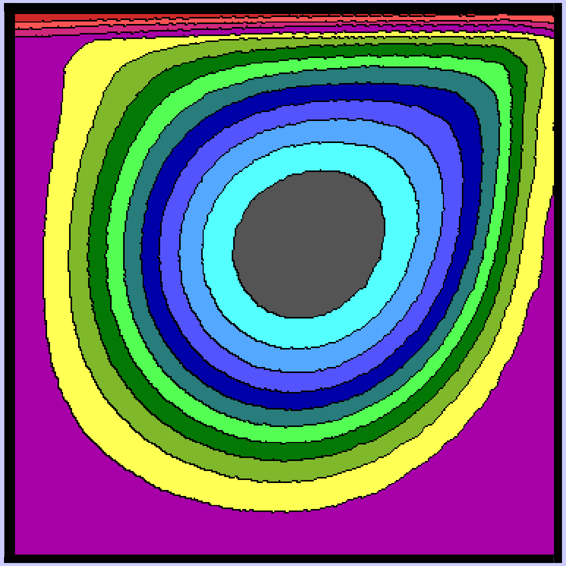


Metoda 4

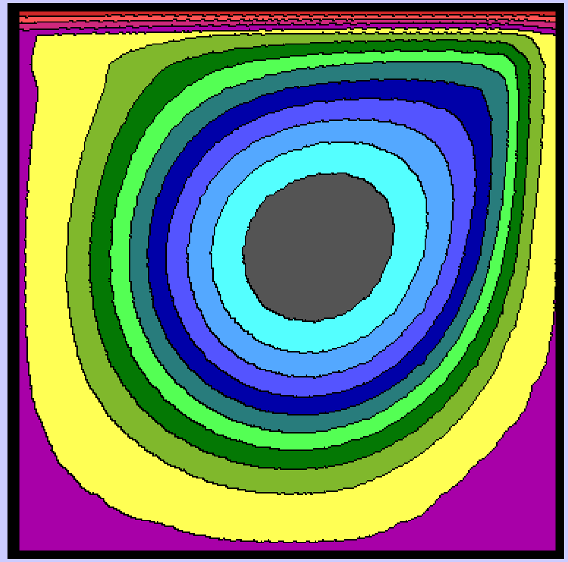


Strujnice u pokretnoj šupljini pri R_e=400

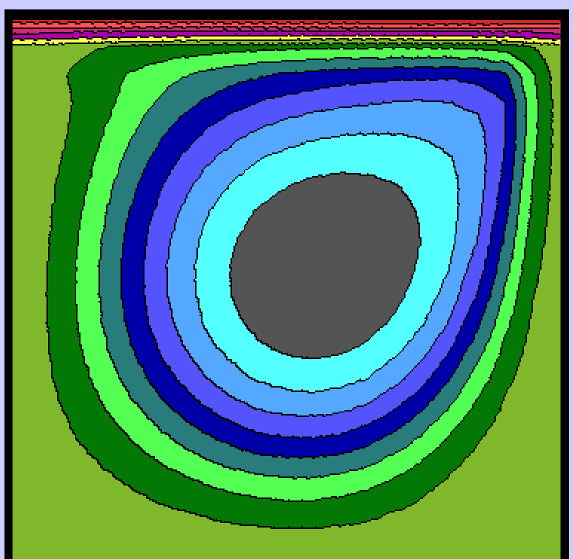
Metoda 1



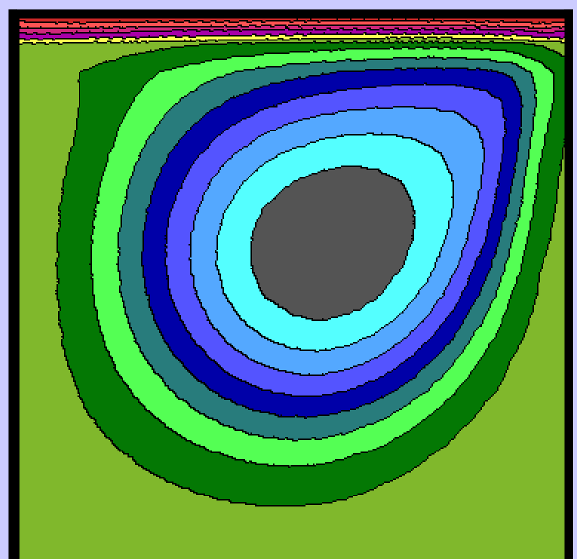
Metoda 2



Metoda 3

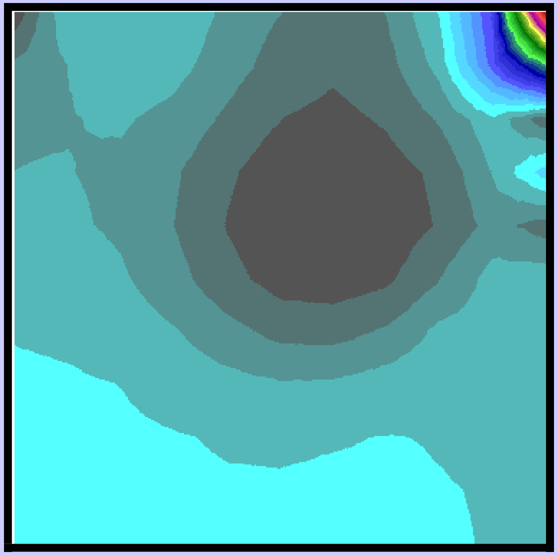


Metoda 4

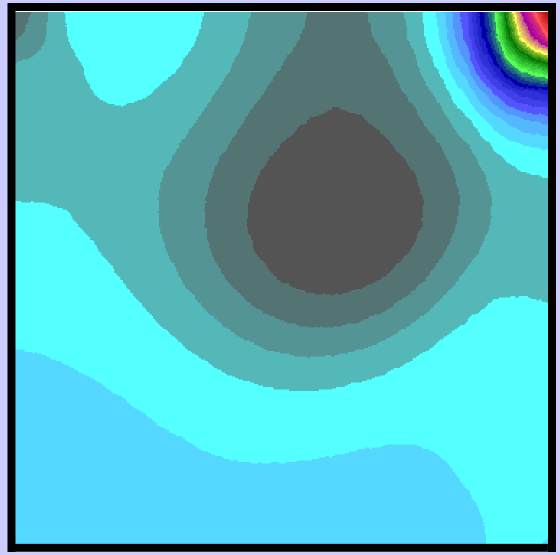


Strujnice u pokretnoj šupljini pri R_e=1000

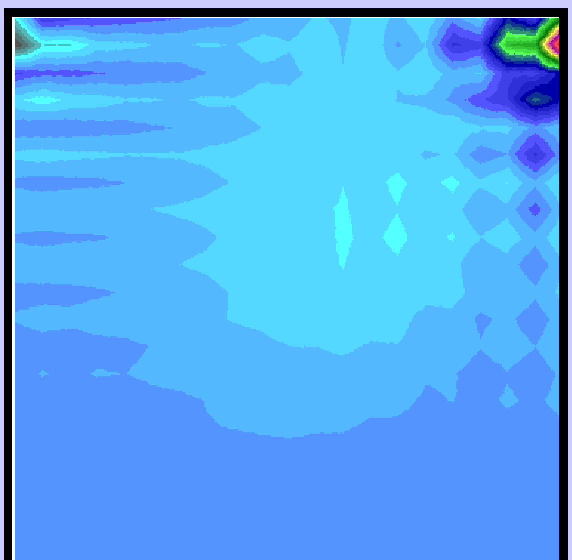
Metoda 1



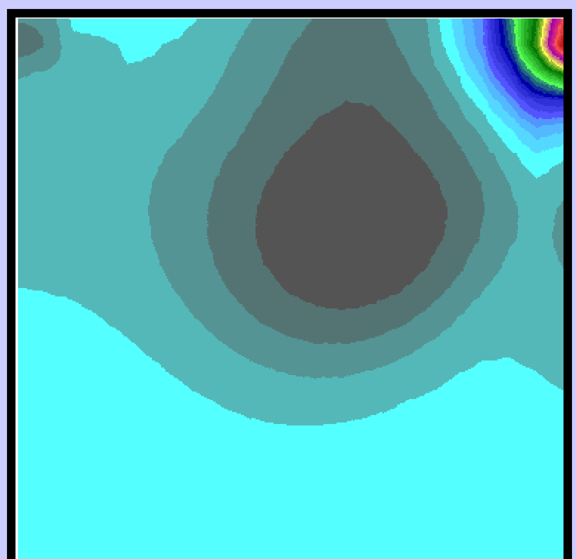
Metoda 2



Metoda 3

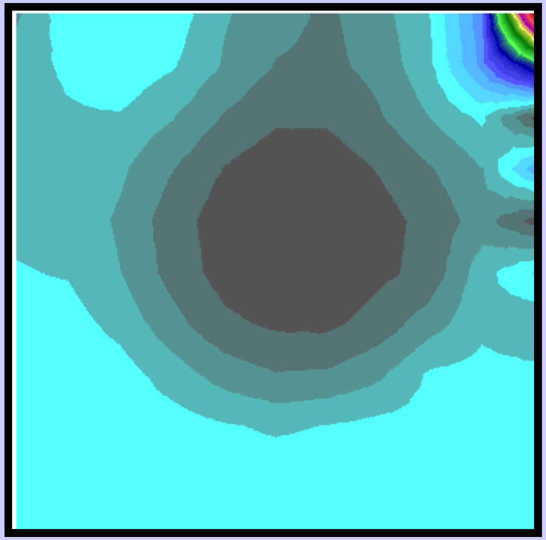


Metoda 4

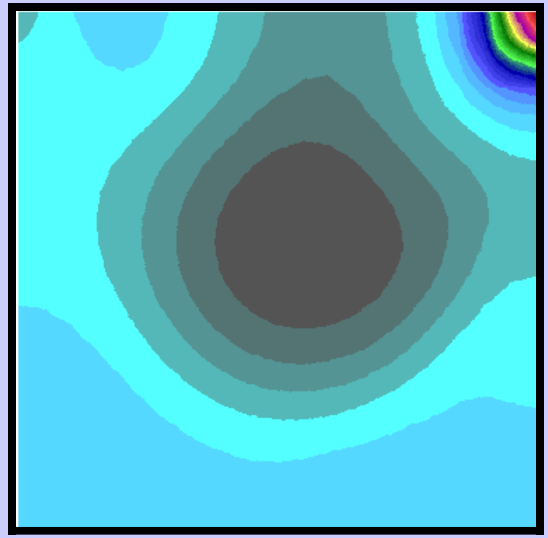


Raspored pritiska u pokretnoj šupljini pri R_e=400

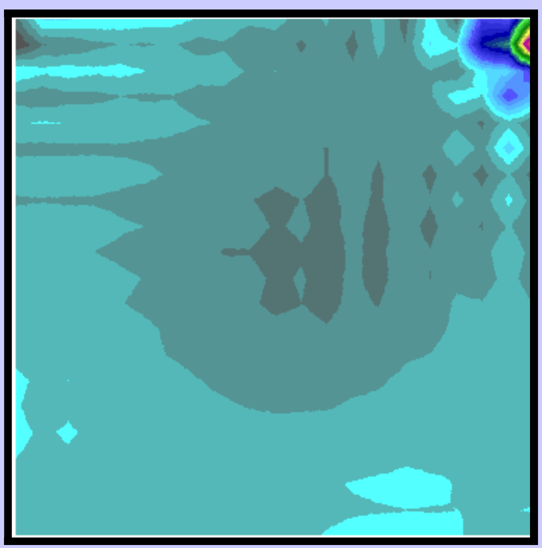
Metoda 1



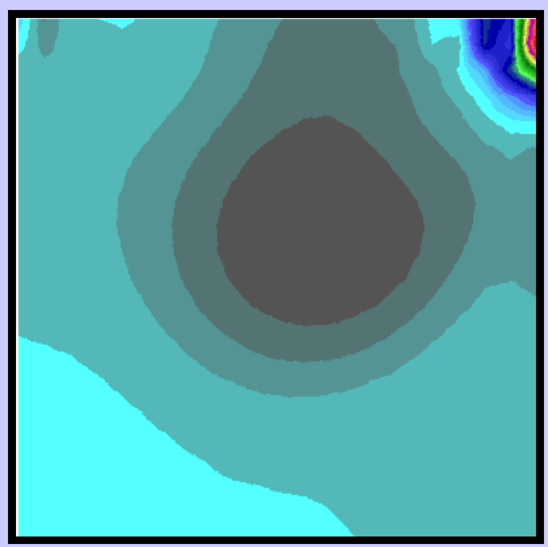
Metoda 2



Metoda 3

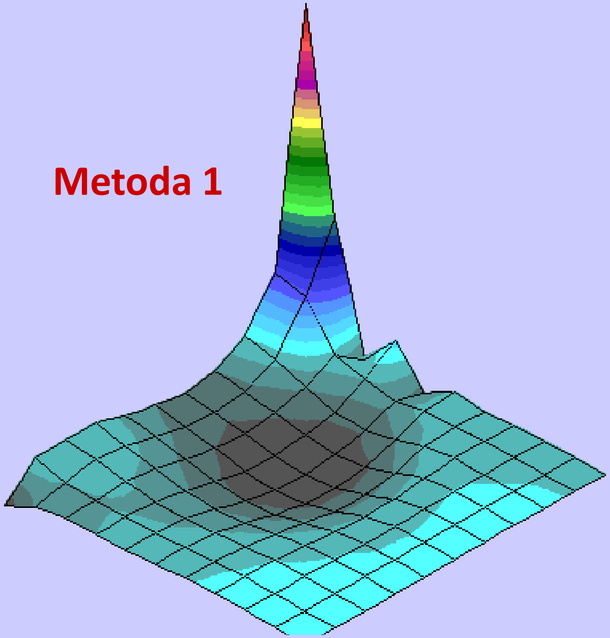


Metoda 4

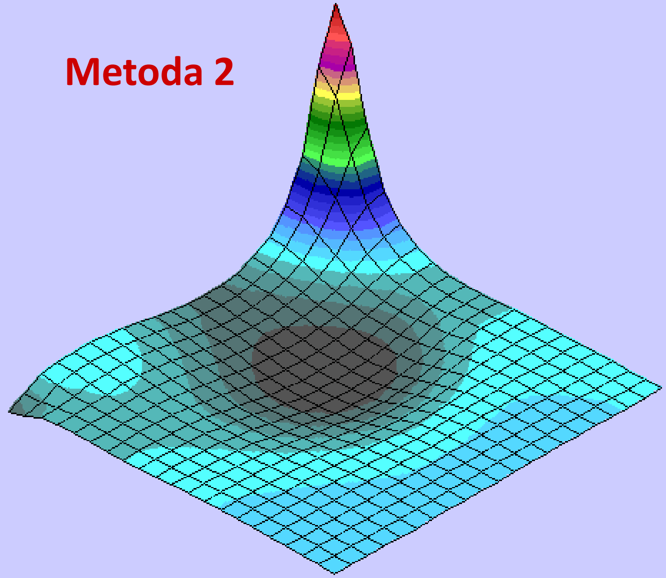


Rapored pritiska u pokretnoj šupljini pri R_e=1000

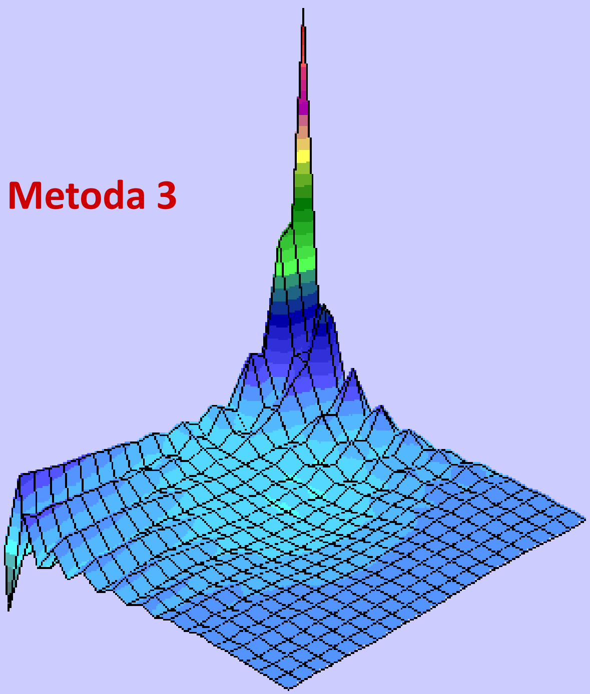
Metoda 1



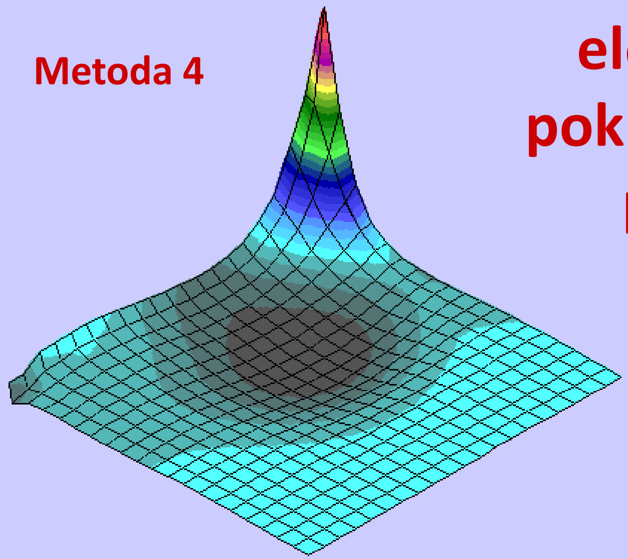
Metoda 2



Metoda 3



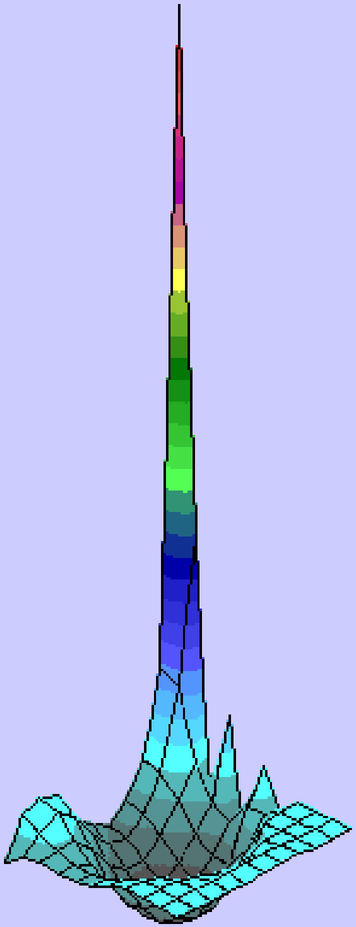
Metoda 4



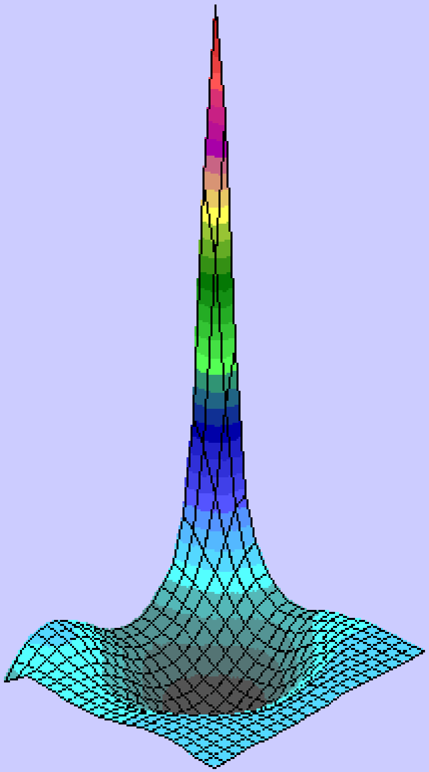
Polja pritiska sa elevacijama u pokretnoj šupljini pri R_e=400

Polja pritiska sa elevacijama u pokretnoj šupljini pri R_e=1000

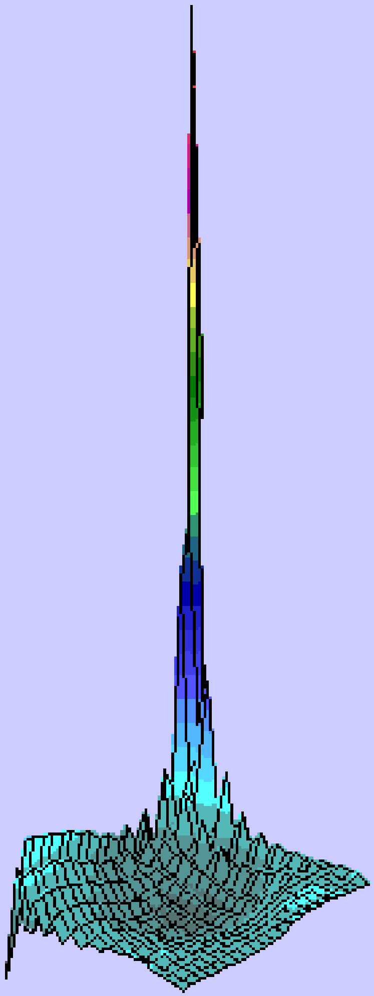
Metoda 1



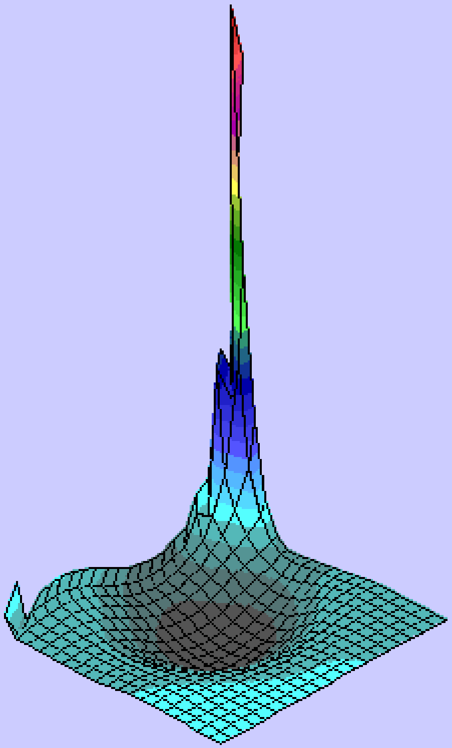
Metoda 2



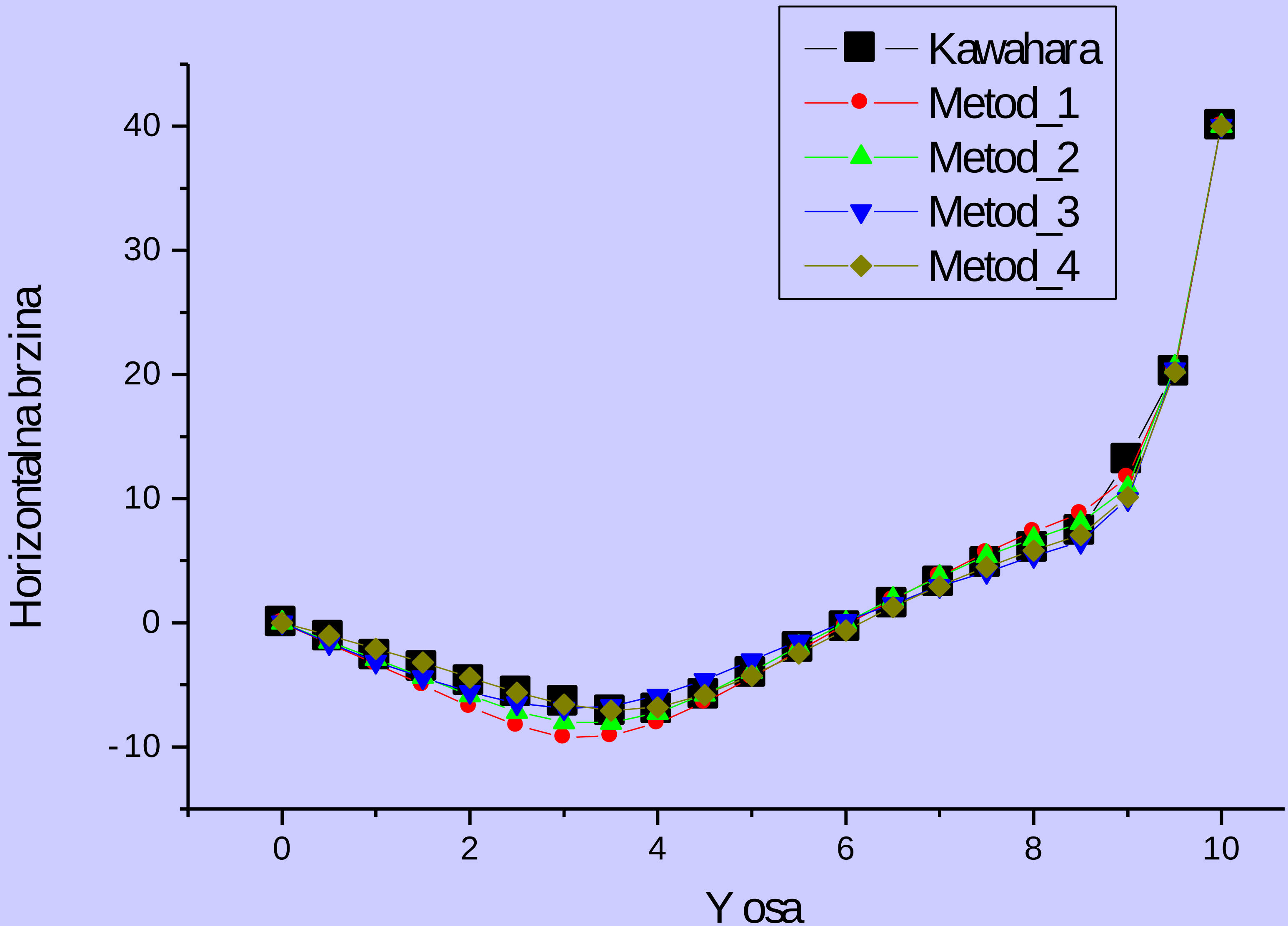
Metoda 3



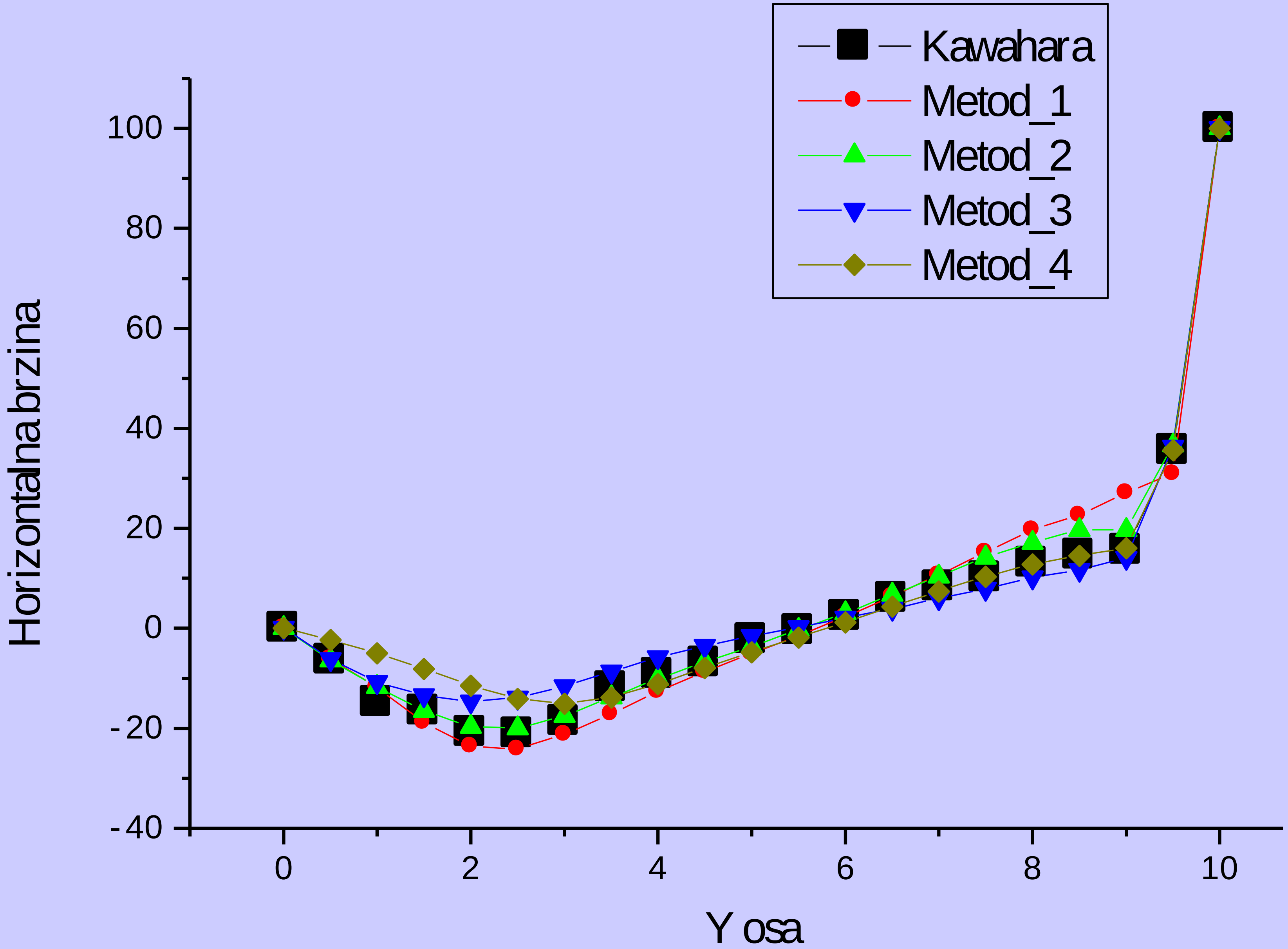
Metoda 4



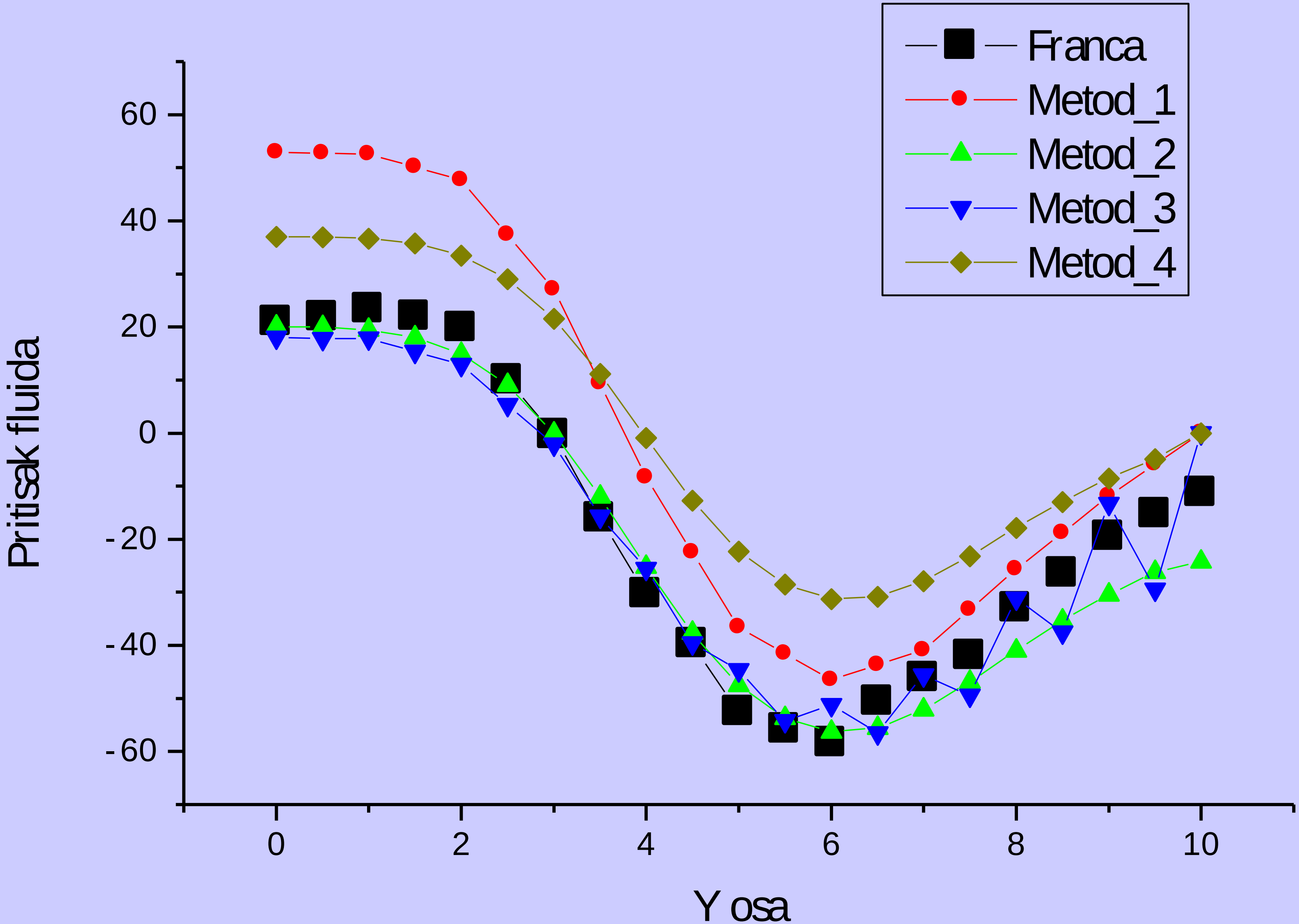
Dijagrami raspodele horizontalne brzine za x=a/2 pri R_e=400



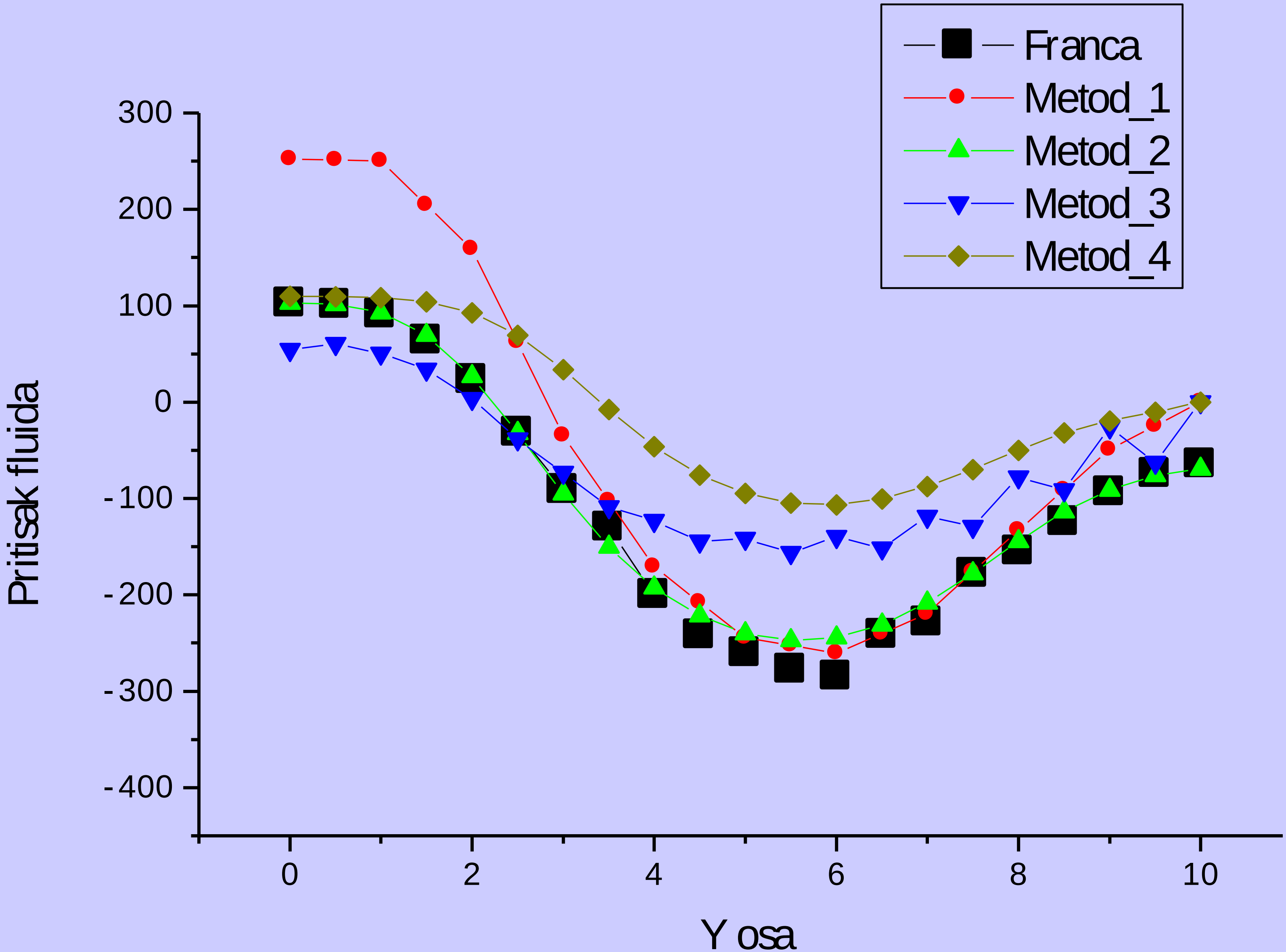
Dijagrami raspodele horizontalne brzine za x=a/2 pri R_e=1000



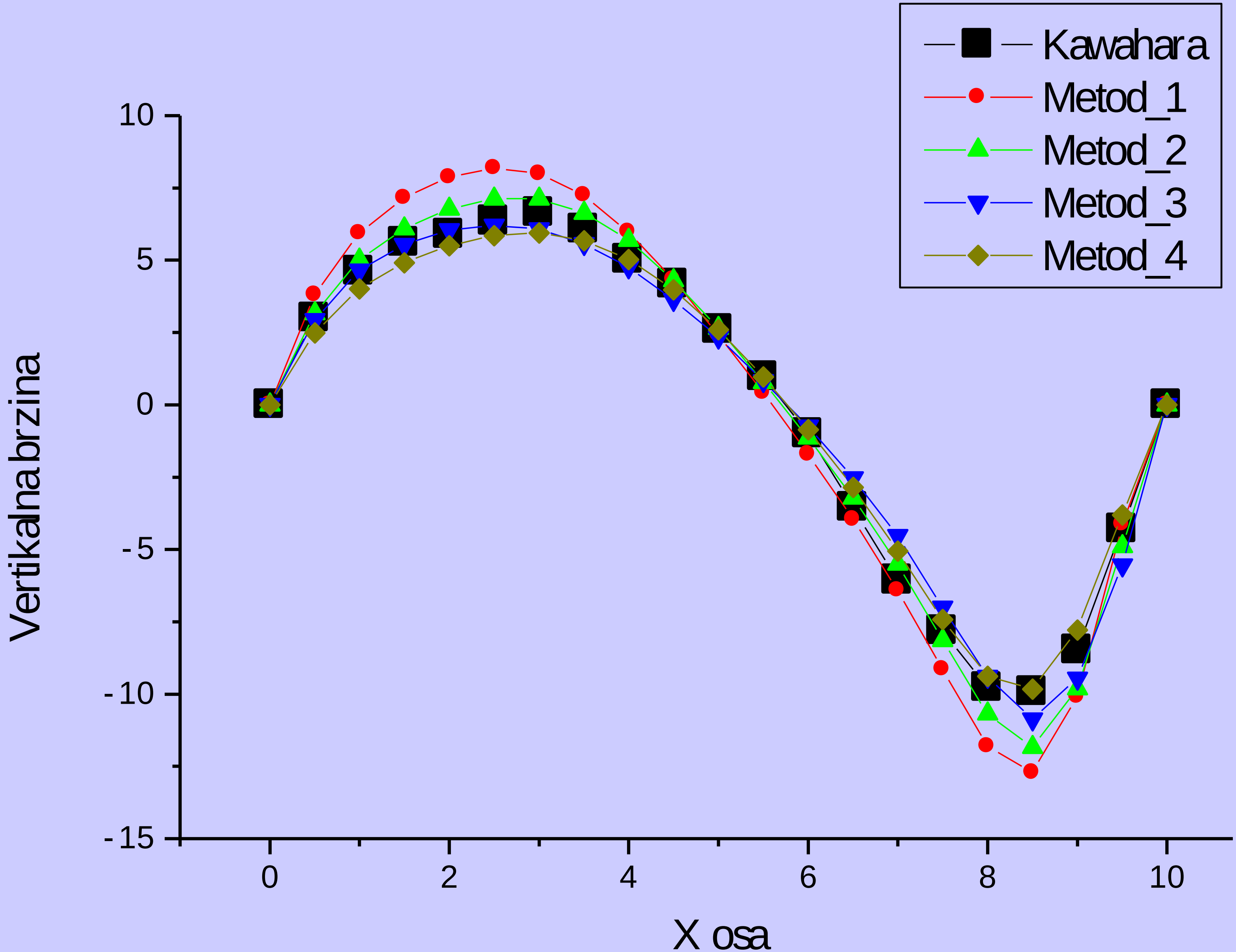
Dijagrami raspodele pritiska za x=a/2 pri R_e=400



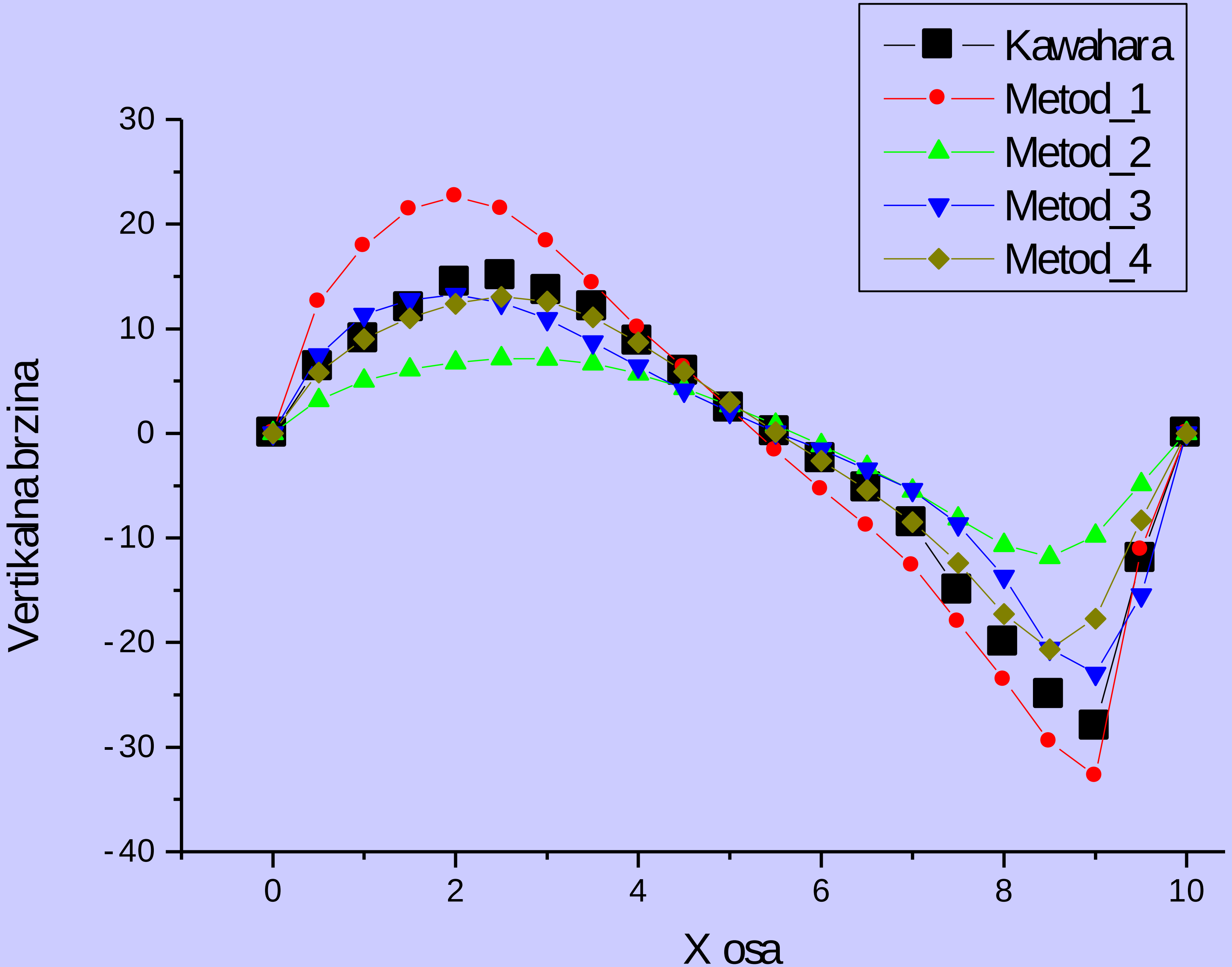
Dijagrami raspodele pritiska za x=a/2 pri R_e=1000



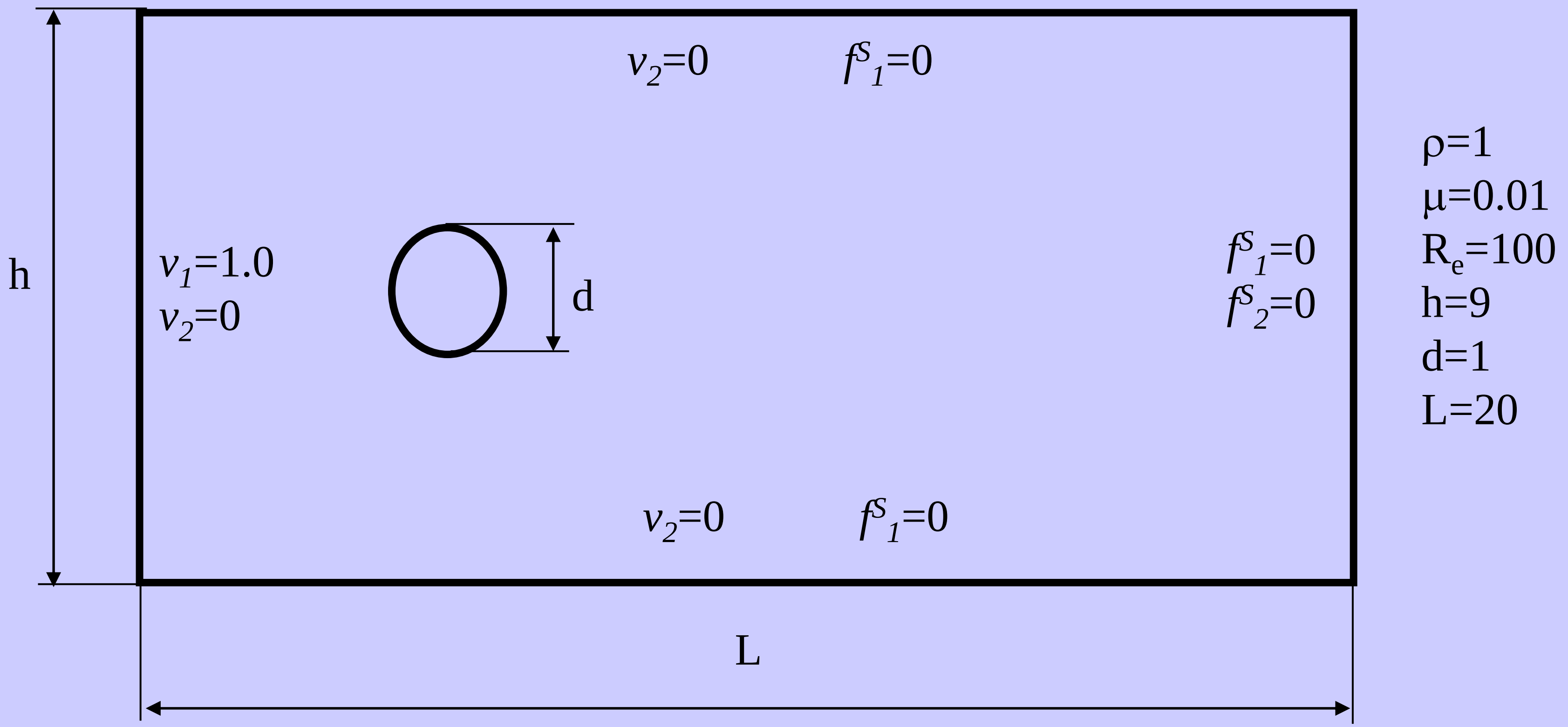
Dijagrami raspodele vertikalne brzine za x=a/2 pri R_e=400



Dijagrami raspodele vertikalne brzine za x=a/2 pri R_e=1000

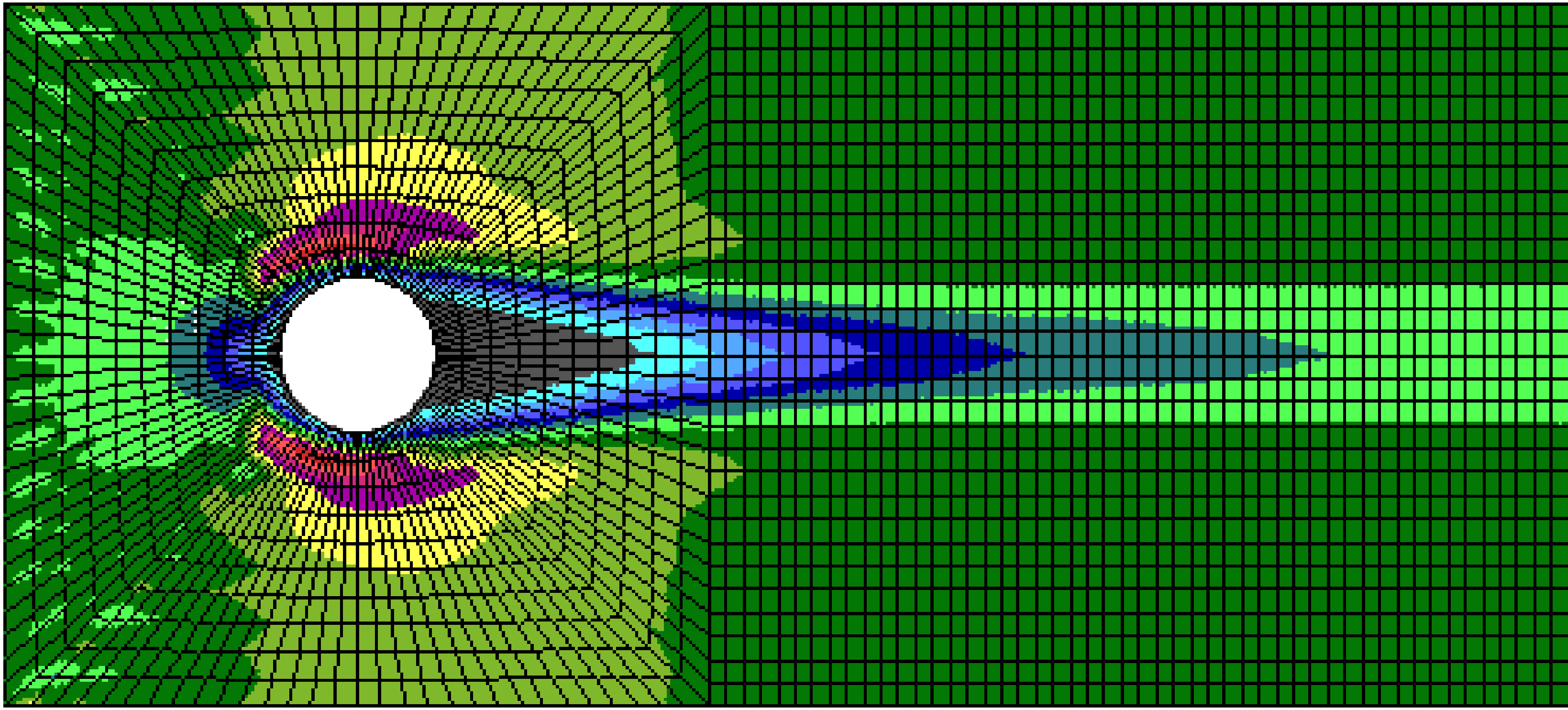


2.8.4 Opstrujavanje cilindra

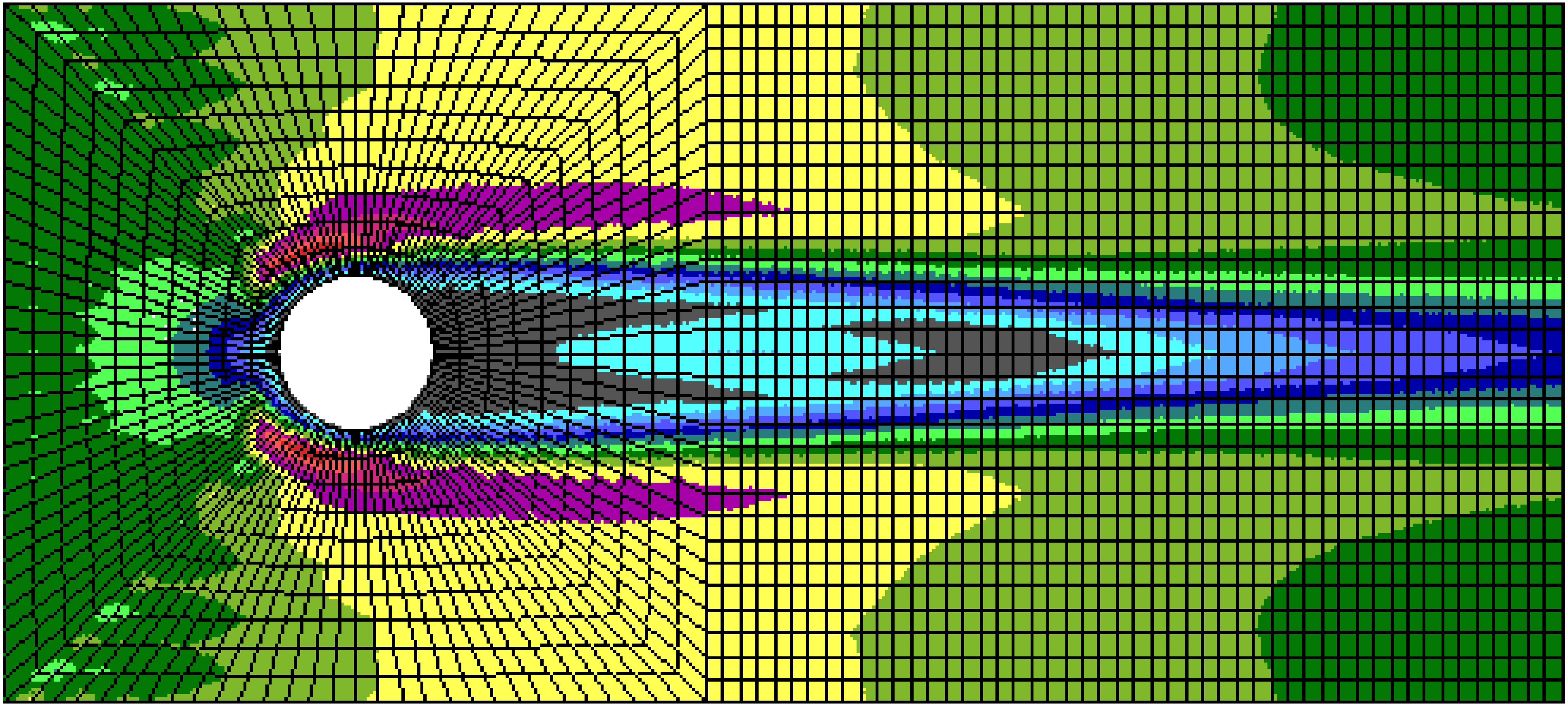


- **'Karman vortex street' vrtlozi**
- **PENALTI metoda sa UPWIND stabilizacionom tehnikom**

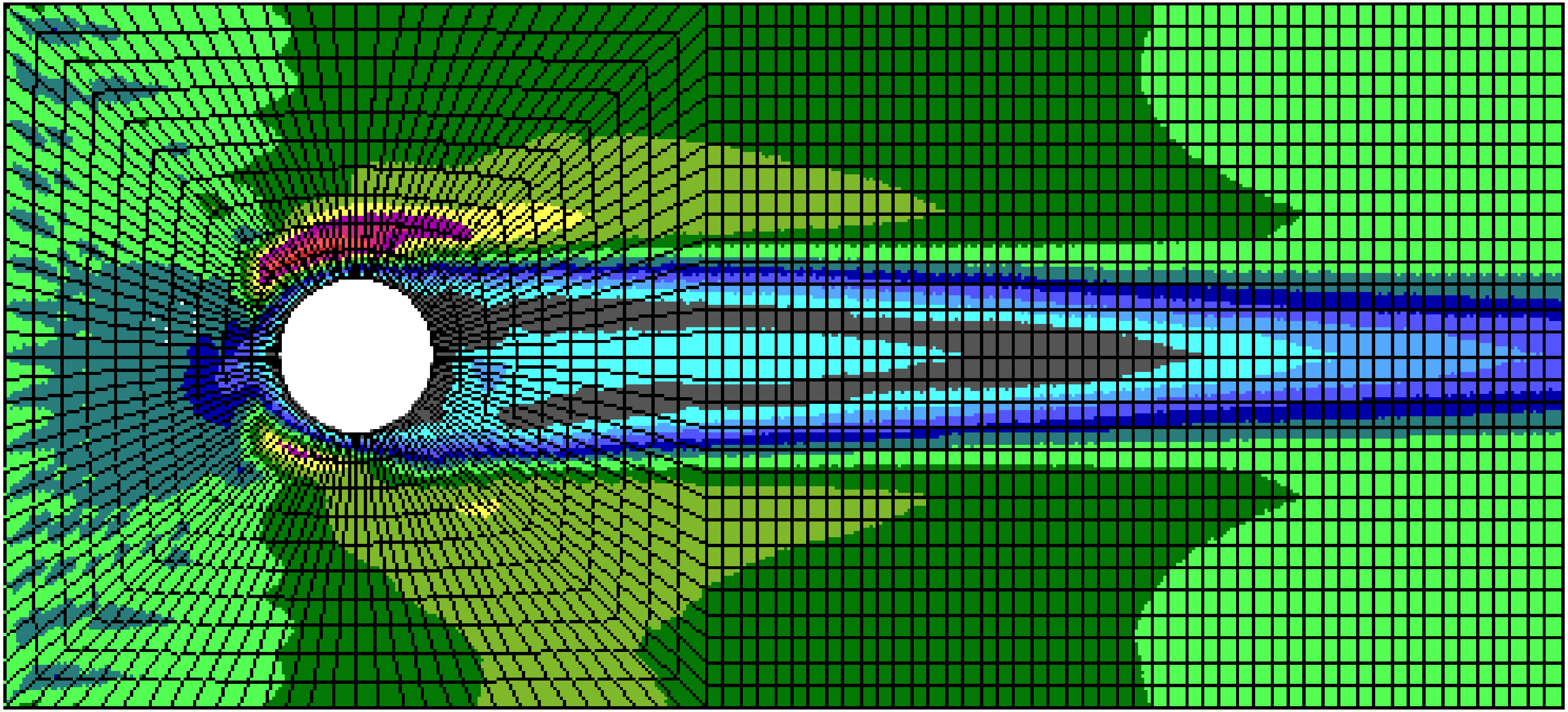
Polje brzine fluida pri opstrujavanju cilindra za t=12 s



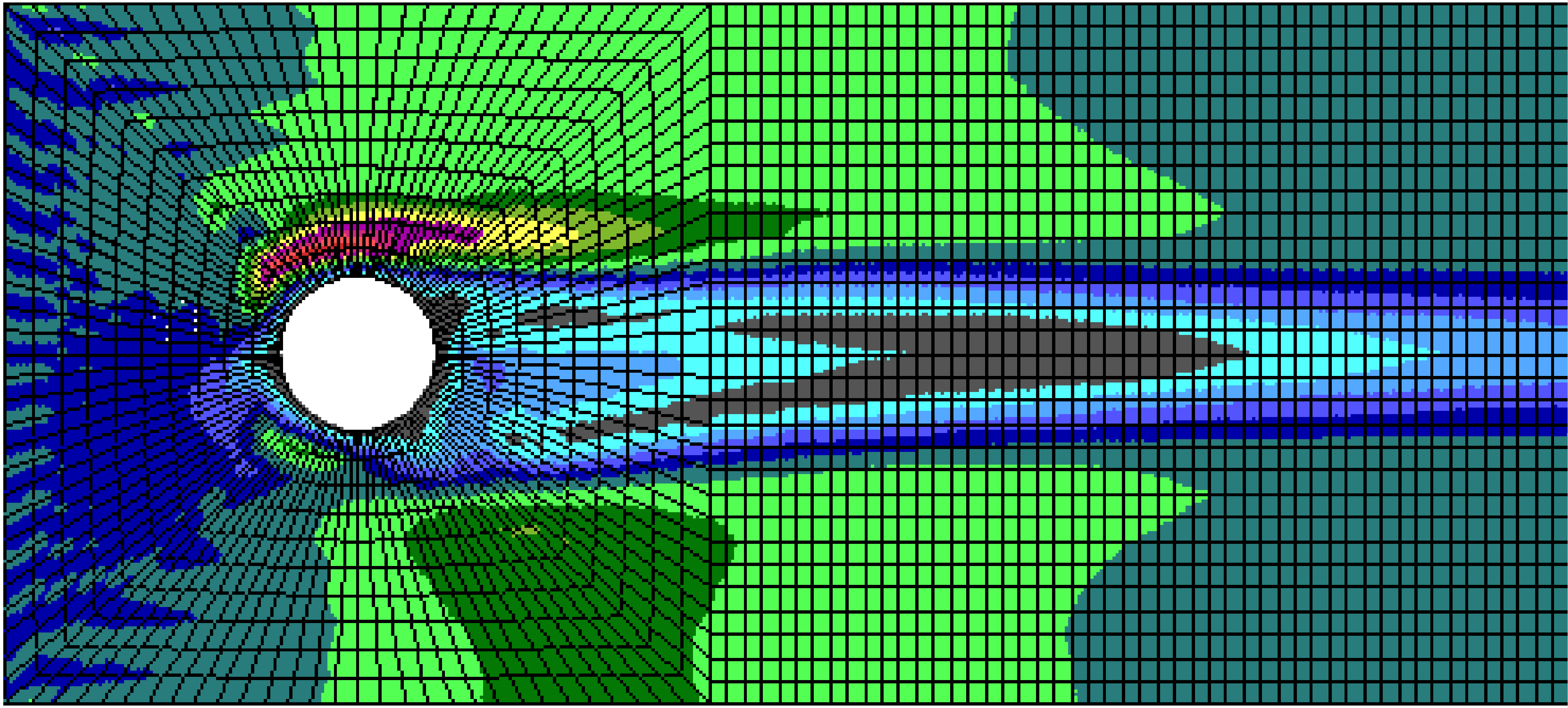
Polje brzine fluida pri opstrujavanju cilindra za t=48 s



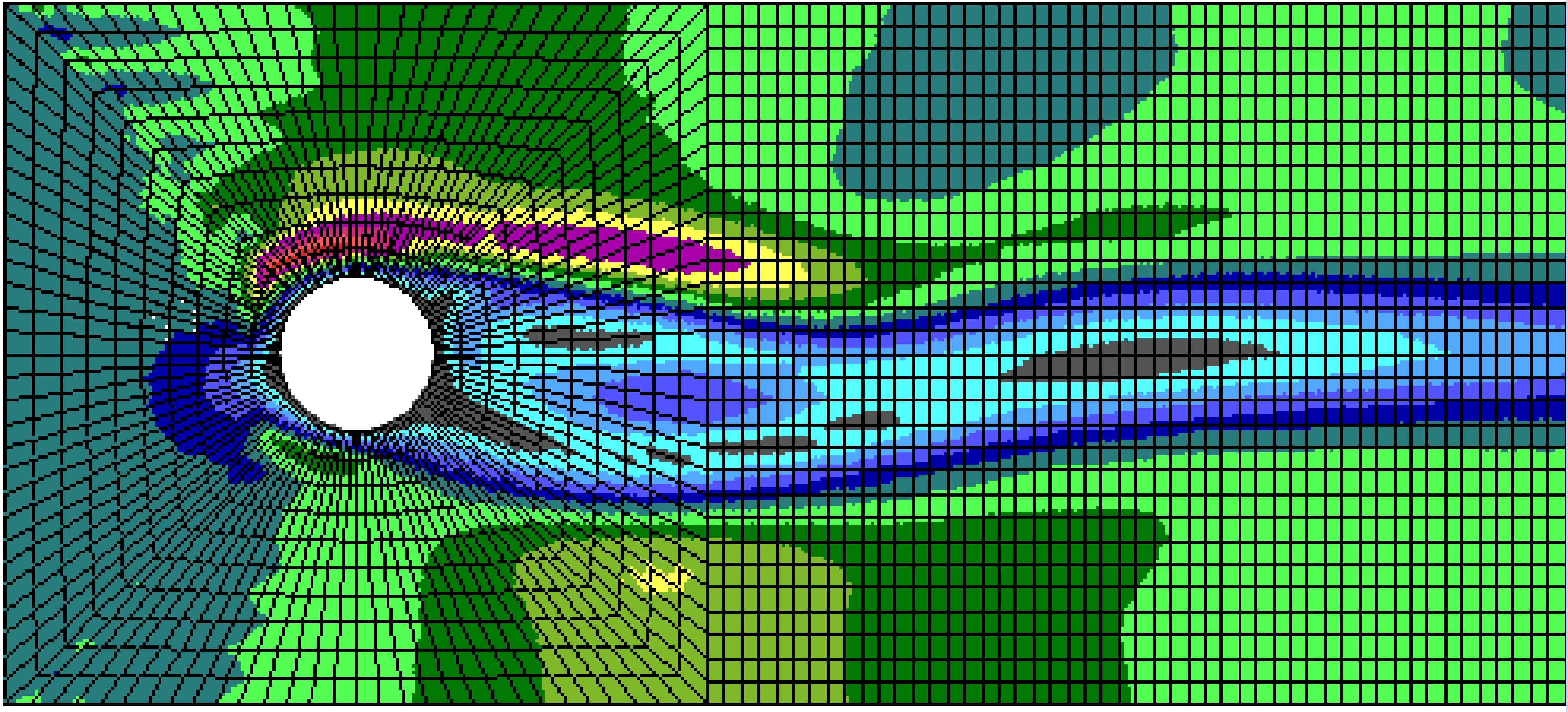
Polje brzine fluida pri opstrujavanju cilindra za t=96 s



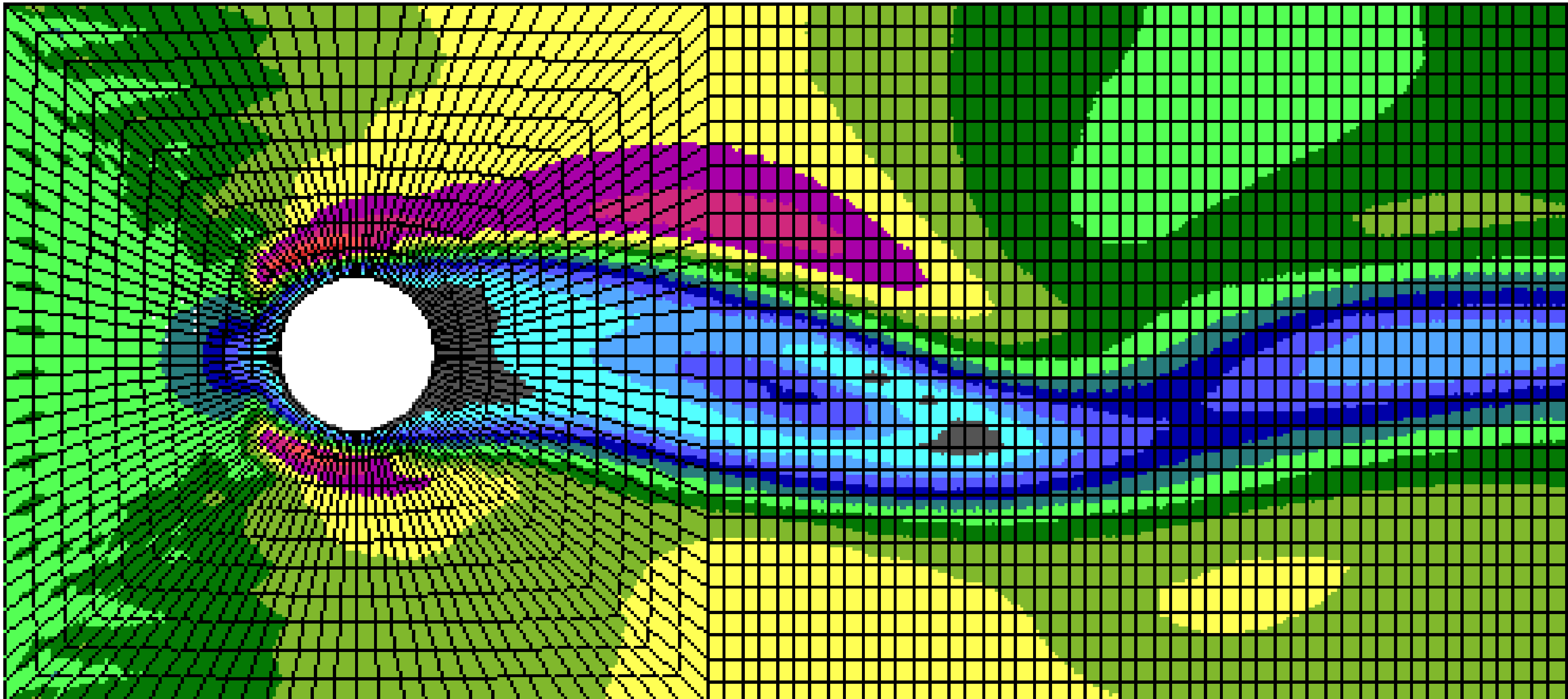
Polje brzine fluida pri opstrujavanju cilindra za t=98 s



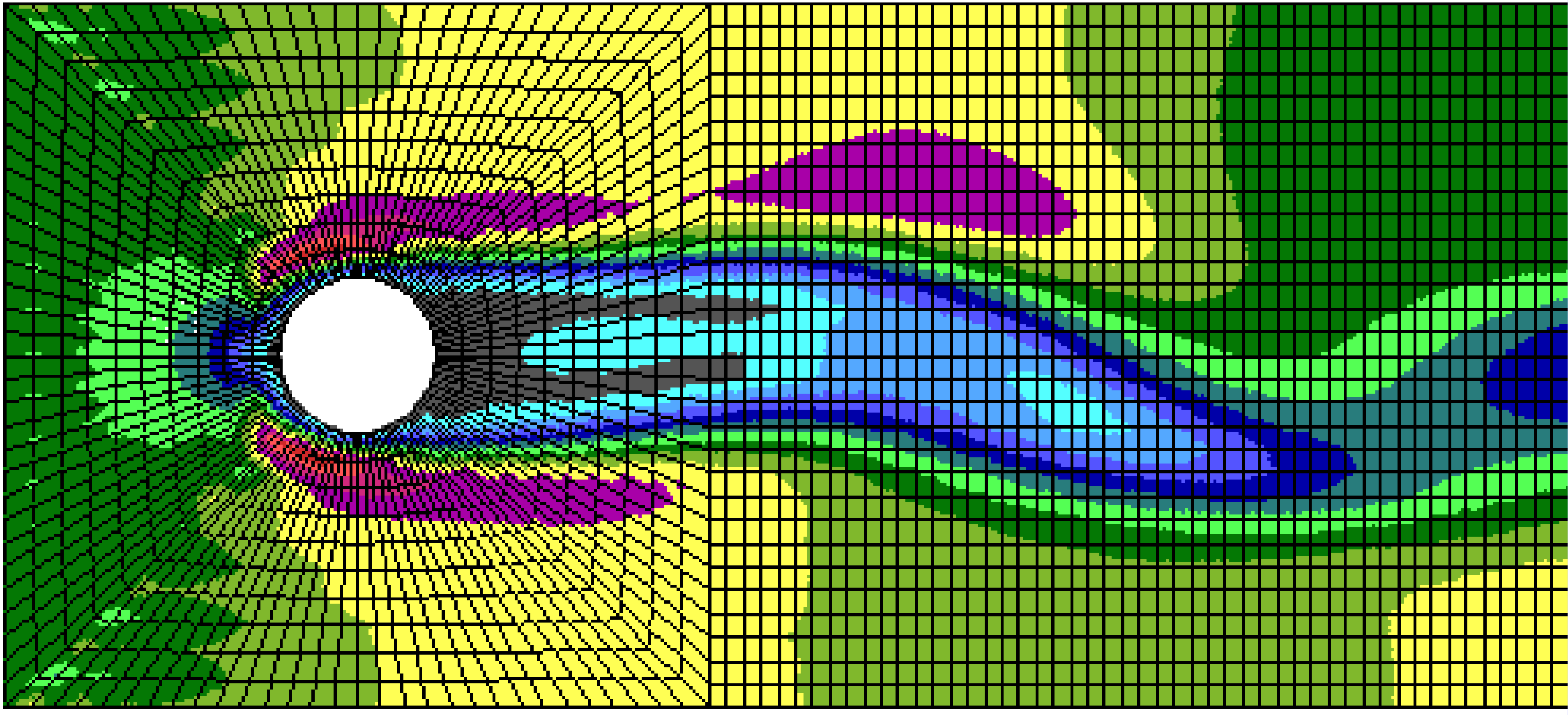
Polje brzine fluida pri opstrujavanju cilindra za t=102 s



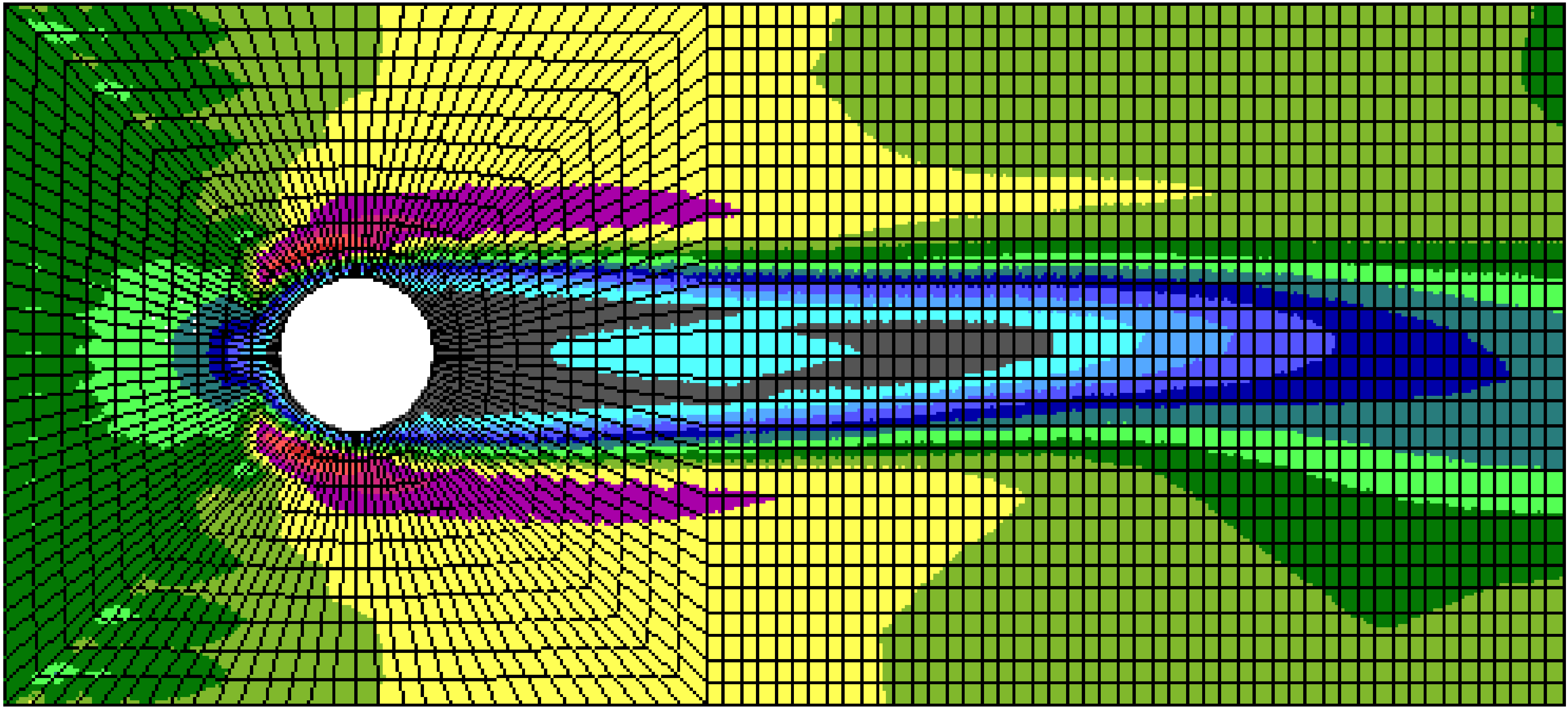
Polje brzine fluida pri opstrujavanju cilindra za t=104 s



Polje brzine fluida pri opstrujavanju cilindra za t=106 s



Polje brzine fluida pri opstrujavanju cilindra za t=132 s



SOLID-FLUID INTERAKCIJA

4.2 OSNOVNE JEDNAČINE SPREZANJA

Inkrementalno-iterativni oblik diferencijalne jedna~ine kretanja nelinearne strukture

$$\mathbf{M}^{t+\Delta t}\ddot{\mathbf{U}}^{(i)}+\mathbf{C}\Delta\dot{\mathbf{U}}^{(i-1)}+{}^{t+\Delta t}\mathbf{K}^{(i-1)}\Delta\mathbf{U}^{(i)}={}^{t+\Delta t}\mathbf{F}_s-\mathbf{C}^{t+\Delta t}\dot{\mathbf{U}}^{(i-1)}-{}^{t+\Delta t}\mathbf{F}^{(i-1)} \qquad (4.2.2)$$

Definisanje ukupnog vektora brzina i ukupnog pomeranja

$$^{t+\Delta t}\dot{\mathbf{U}}^{(i)}={}^{t}\dot{\mathbf{U}}+{}^{t}\Delta\dot{\mathbf{U}}^{(i-1)}+\Delta\dot{\mathbf{U}}^{(i)}={}^{t+\Delta t}\dot{\mathbf{U}}^{(i-1)}+\Delta\dot{\mathbf{U}}^{(i)}$$

$$^{t+\Delta t}\mathbf{U}^{(i)}={}^{t+\Delta t}\mathbf{U}^{(i-1)}+\Delta\mathbf{U}^{(i)} \qquad (4.2.3)$$

Jedna~ina solida po nepoznatim brzinama

$$^{t+\Delta t}\hat{\mathbf{C}}^{(i-1)}\Delta\dot{\mathbf{U}}^{(i)}={}^{t+\Delta t}\hat{\mathbf{F}}_s^{(i-1)} \qquad (4.2.9)$$

$$^{t+\Delta t}\hat{\mathbf{C}}^{(i-1)}=b_0\mathbf{M}+\mathbf{C}+b_1{}^{t+\Delta t}\mathbf{K}^{(i-1)} \qquad (4.2.10)$$

$$^{t+\Delta t}\hat{\mathbf{F}}_s^{(i-1)}={}^{t+\Delta t}\mathbf{F}_s-\mathbf{M}^{t+\Delta t}\hat{\ddot{\mathbf{U}}}^{(i-1)}-\mathbf{C}^{t+\Delta t}\dot{\mathbf{U}}^{(i-1)}-{}^{t+\Delta t}\mathbf{K}^{(i-1)}{}^{t+\Delta t}\mathbf{U}^{(i-1)}-{}^{t+\Delta t}\mathbf{F}^{(i-1)} \qquad (4.2.11)$$

$$^{t+\Delta t}\hat{\ddot{\mathbf{U}}}^{(i-1)}=b_0\left({}^{t+\Delta t}\dot{\mathbf{U}}^{(i-1)}-{}^{t}\dot{\mathbf{U}}\right)+b_2{}^{t}\ddot{\mathbf{U}} \qquad (4.2.12)$$

$$^{t+\Delta t}\mathbf{U}^{(i-1)}={}^{t}\mathbf{U}+b_3{}^{t}\dot{\mathbf{U}}+b_4{}^{t}\ddot{\mathbf{U}}+b_5{}^{t+\Delta t}\dot{\mathbf{U}}^{(i-1)} \qquad (4.2.13)$$

Posle postizanja konvergencije ukupni vektori ubrzanja i pomeranja se ra~unaju prema jedna~inama

$$ {}^{t+\Delta t}\ddot{\mathbf{U}}^{(i)} = b_0\left({}^{t+\Delta t}\dot{\mathbf{U}}^{(i)} - {}^{t}\dot{\mathbf{U}}\right) + b_2\,{}^{t}\ddot{\mathbf{U}} \quad (4.2.14) $$

$$ {}^{t+\Delta t}\mathbf{U}^{(i)} = {}^{t}\mathbf{U} + b_6\,{}^{t}\dot{\mathbf{U}} + b_7\,{}^{t}\ddot{\mathbf{U}} + b_8\,{}^{t+\Delta t}\ddot{\mathbf{U}}^{(i)} \quad (4.2.15) $$

Sistem jedna~ina za solid

$$ \begin{bmatrix} \mathbf{K}^{s}_{vf-f} & \mathbf{K}^{s}_{vf-s} \\ \mathbf{K}^{s}_{vs-f} & \mathbf{K}^{s}_{vs-s} \end{bmatrix} \begin{pmatrix} \Delta\mathbf{v}_{sf} \\ \Delta\mathbf{v}_{ss} \end{pmatrix} = \begin{pmatrix} \mathbf{f}_{sf} \\ \mathbf{f}_{ss} \end{pmatrix} + \begin{pmatrix} \tilde{\mathbf{f}}_{fs} \\ \mathbf{0} \end{pmatrix} \quad (4.2.19) $$

Sistem jedna~ina za fluid

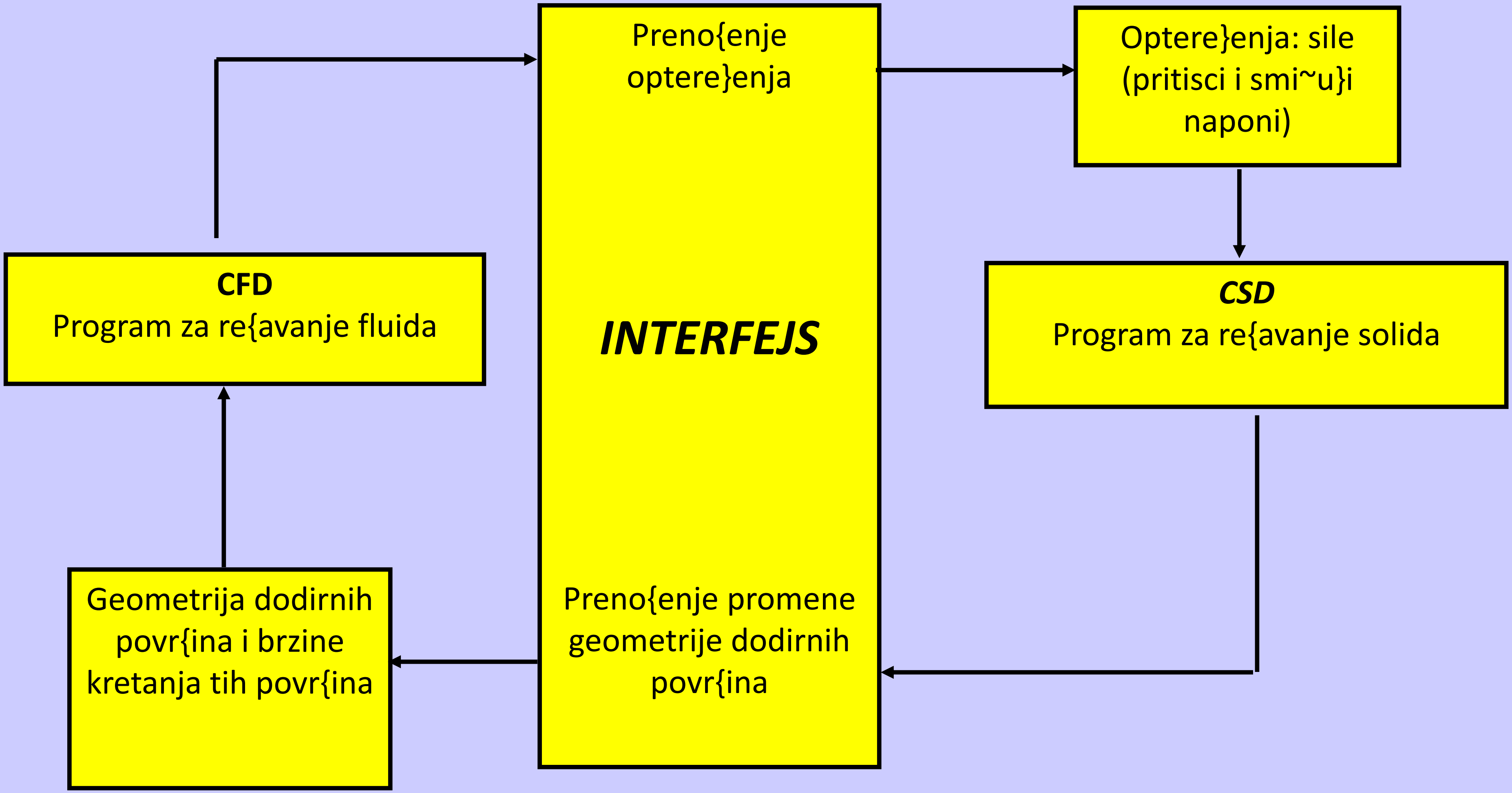
$$ \begin{bmatrix} \mathbf{K}^{f}_{vs-s} & \mathbf{K}^{f}_{vs-f} & \mathbf{K}^{f}_{ps-p} \\ \mathbf{K}^{f}_{vf-s} & \mathbf{K}^{f}_{vf-f} & \mathbf{K}^{f}_{pf-p} \\ \mathbf{K}^{\mathrm{T}f}_{ps-p} & \mathbf{K}^{\mathrm{T}f}_{pf-p} & \mathbf{0} \end{bmatrix} \begin{pmatrix} \Delta\mathbf{v}_{fs} \\ \Delta\mathbf{v}_{ff} \\ \Delta p \end{pmatrix} = \begin{pmatrix} \mathbf{f}_{\mathbf{v}_{fs}} \\ \mathbf{f}_{\mathbf{v}_{ff}} \\ \mathbf{f}_{p} \end{pmatrix} \quad (4.2.20) $$

Spregnuti sistem jedna~ina

$$ \begin{bmatrix} \mathbf{K}^{f}_{vs-s} + \mathbf{K}^{s}_{vf-f} & \mathbf{K}^{f}_{vs-f} & \mathbf{K}^{f}_{ps-p} & \mathbf{K}^{s}_{vf-s} \\ \mathbf{K}^{f}_{vf-s} & \mathbf{K}^{f}_{vf-f} & \mathbf{K}^{f}_{pf-p} & \mathbf{0} \\ \mathbf{K}^{\mathrm{T}f}_{ps-p} & \mathbf{K}^{\mathrm{T}f}_{pf-p} & \mathbf{0} & \mathbf{0} \\ \mathbf{K}^{s}_{vs-f} & \mathbf{0} & \mathbf{0} & \mathbf{K}^{f}_{vs-s} \end{bmatrix} \begin{pmatrix} \Delta\mathbf{v}_{fs} \\ \Delta\mathbf{v}_{ff} \\ \Delta p \\ \Delta\mathbf{v}_{s} \end{pmatrix} = \begin{pmatrix} \mathbf{f}_{\mathbf{v}_{fs}} + \mathbf{f}_{sf} \\ \mathbf{f}_{\mathbf{v}_{ff}} \\ \mathbf{f}_{p} \\ \mathbf{f}_{ss} \end{pmatrix} + \begin{pmatrix} \mathbf{0} \\ \tilde{\mathbf{f}}_{fs} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} $$

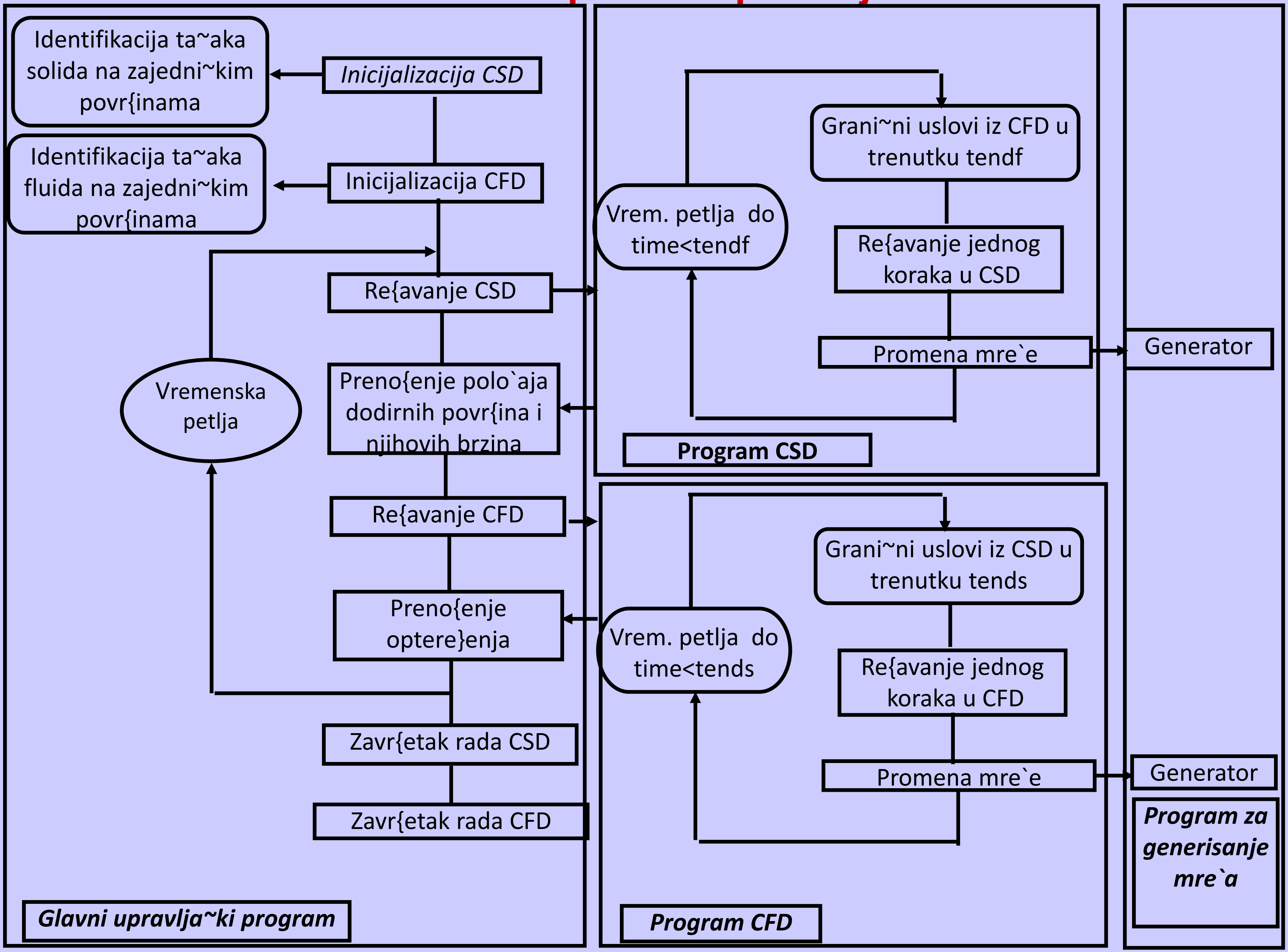
(4.2.21)

4.4 SLABO SPREZANJE



Razmena informacija za re{avanje problema solid-fluid interakcije

4.4.1 Eksplicitno sprezanje



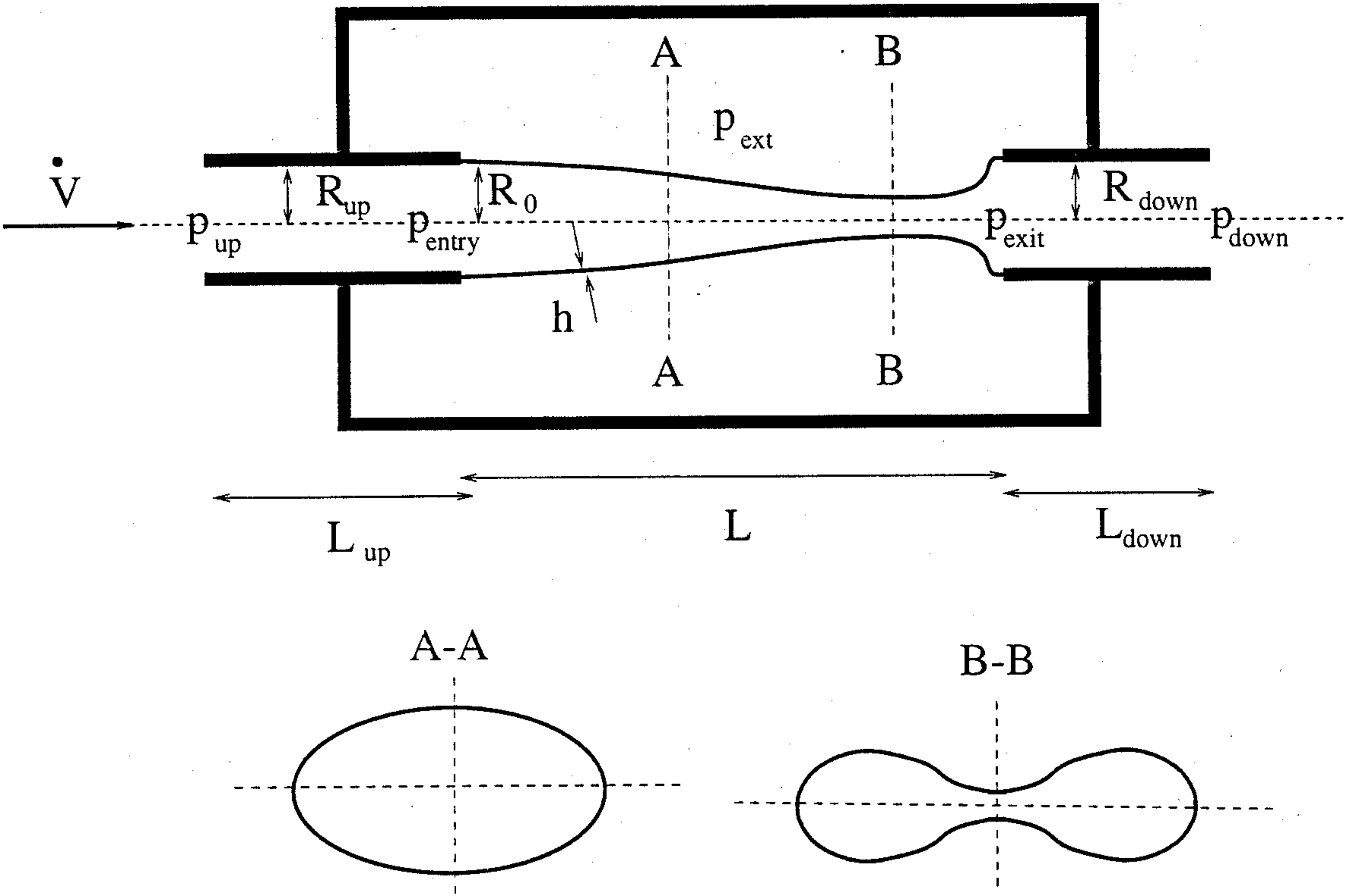
4.4.2 Implicitno sprezanje

1. *Inicijalizuju se re{enja za CFD i CSD*
2. *Startuje se globalna vremenska petlja*
3. *Inicijalizuju se nepoznate veli~ine za vreme n+1:* $\mathbf{x}_0^{n+1} = \mathbf{x}^n, \quad \mathbf{v}_0^{n+1} = \mathbf{v}^n \ \text{ i } \ p^{n+1} = p^n$
4. *i=i+1*
5. *CSD prediktor:* $\tilde{\mathbf{x}}_i^{n+1}, \tilde{\mathbf{v}}_i^{n+1} = f\left(p_{i-1}^{n}\right)$
6. *Prenesu se pretpostavljeni polo`aji i brzine ta~aka na zajedni~kim povr{inama*
7. *CFD prediktor:* $\tilde{p}_i^{n+1} = f\left(\tilde{\mathbf{x}}_i^{n+1}, \tilde{\mathbf{v}}_i^{n+1}\right)$
8. *Prenesu se pretpostavljenja optere}enja od fluida*
9. *CSD korektor:* $\mathbf{x}_i^{n+1}, \mathbf{v}_i^{n+1} = f\left(\tilde{p}_i^{n+1}\right)$
10. *Prenesu se korigovani polo`ji i brzine ta~aka na zajedni~kim povr{inama*
11. *CFD korektor:* $p_i^{n+1} = f\left(\mathbf{x}_i^{n+1}, \mathbf{v}_i^{n+1}\right)$
12. *Prenesu se optere}enja od fluida*
13. *Ako re{enje nije konvergiralo, vratiti se na korak 4 (na slede}u iteraciju)*
14. *Kraj globalne petlje: vratiti se na korak 2 (na slede}i vremenski korak)*

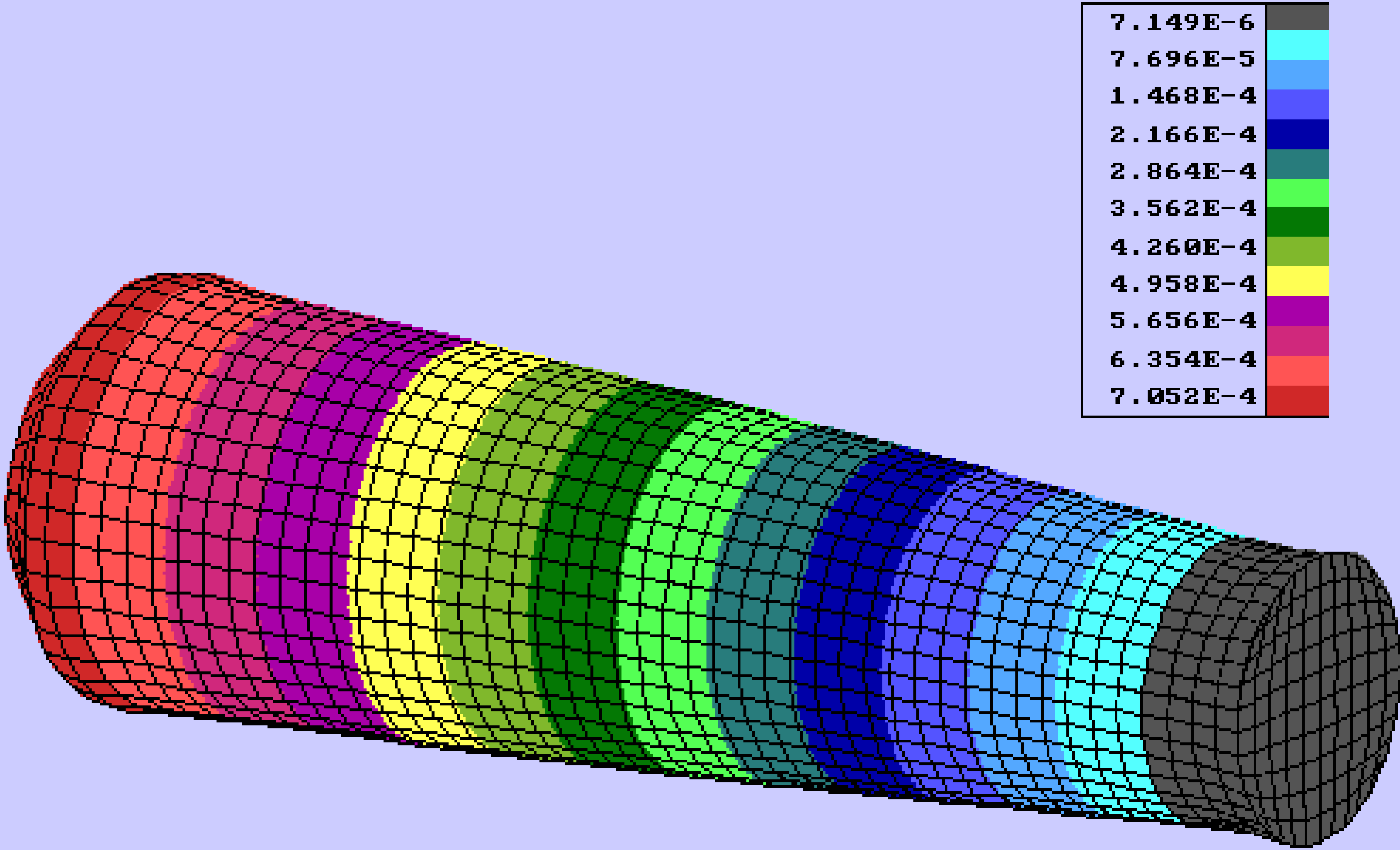
Kriterijum konvergencije

$$\left\|\mathbf{x}_i^{n+1} - \mathbf{x}_{i-1}^{n+1}\right\| < \varepsilon \ \text{ AND } \ \left\|\mathbf{v}_i^{n+1} - \mathbf{v}_{i-1}^{n+1}\right\| < \varepsilon \ \text{ AND } \ \left\|p_i^{n+1} - p_{i-1}^{n+1}\right\| < \varepsilon$$

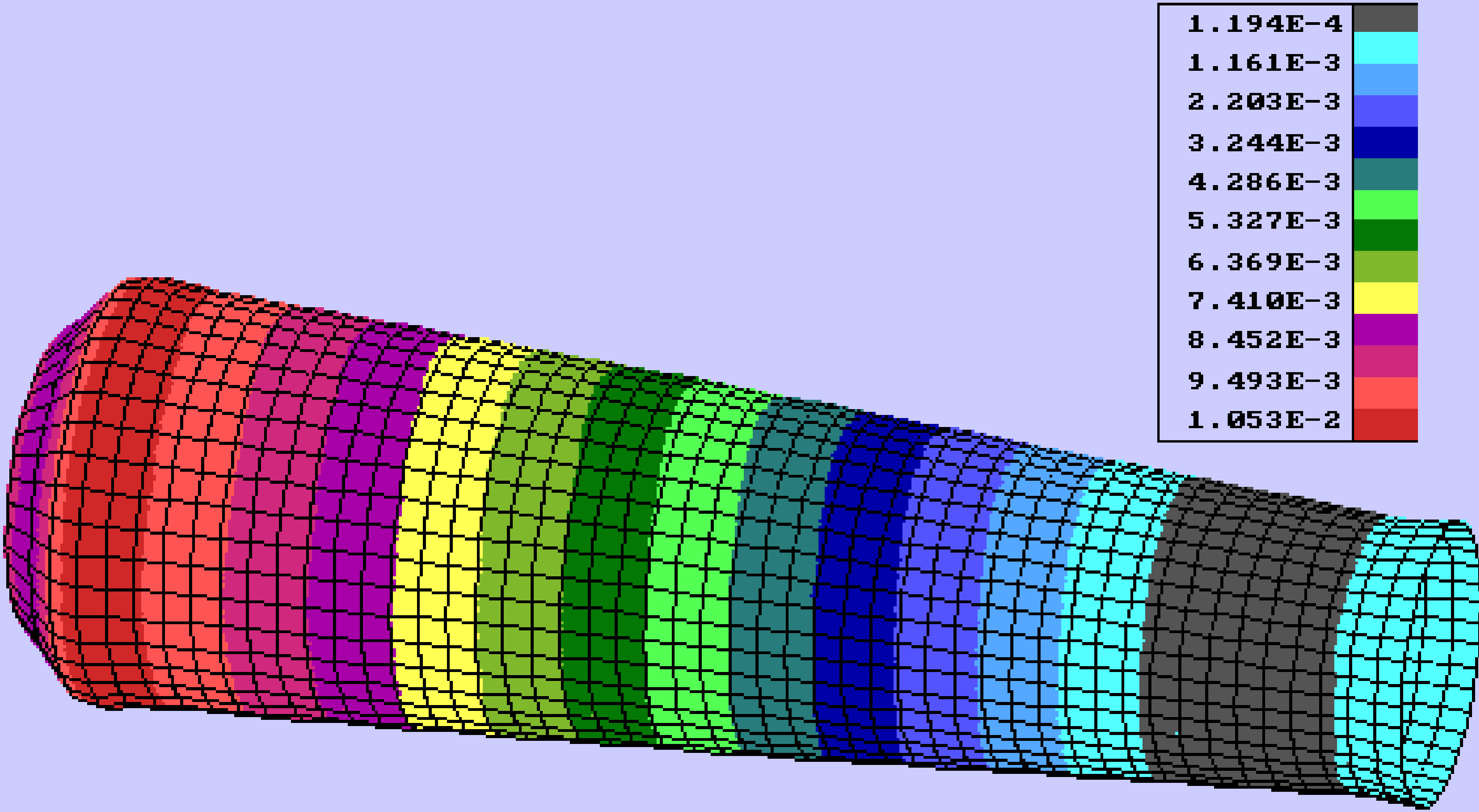
4.5.1 Strujanje fluida u kolapsibilnim cevima



Pritisci fluida u deformisanoj cevi za vreme dejstva pozitivnog transmuralnog pritiska

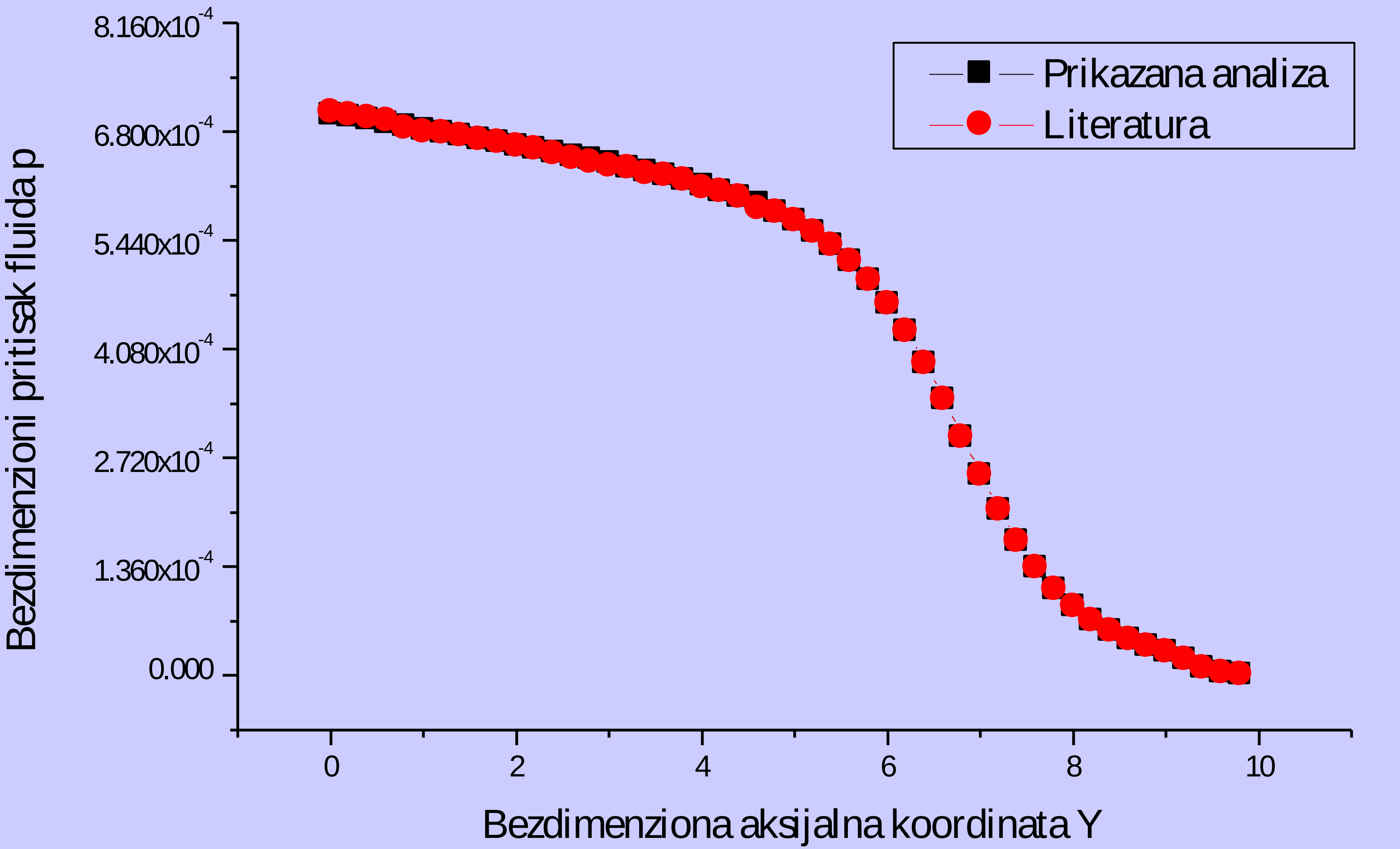


Polje efektivnog napona na zidovima cevi za vreme dejstva pozitivnog transmuralnog pritiska

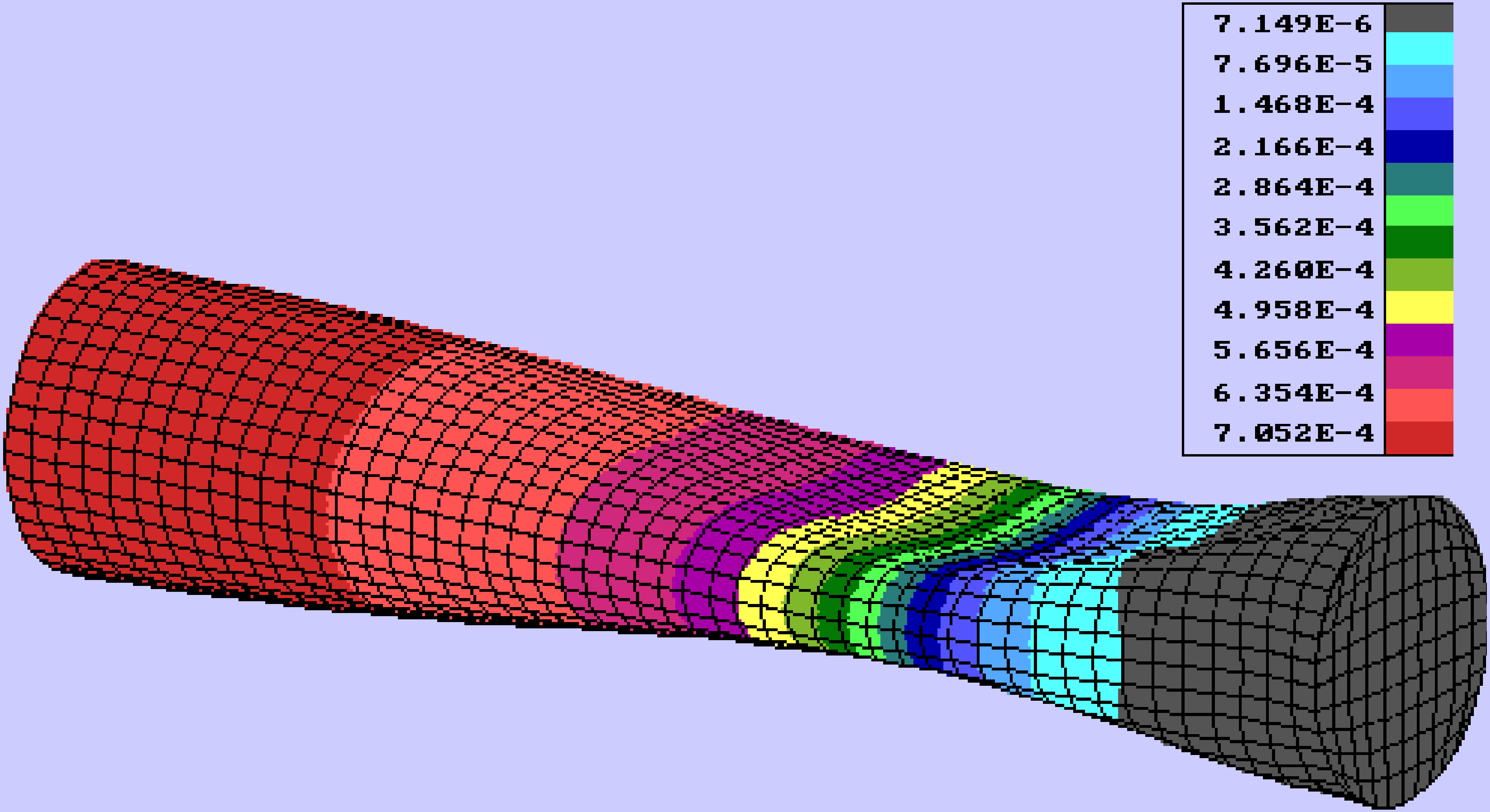


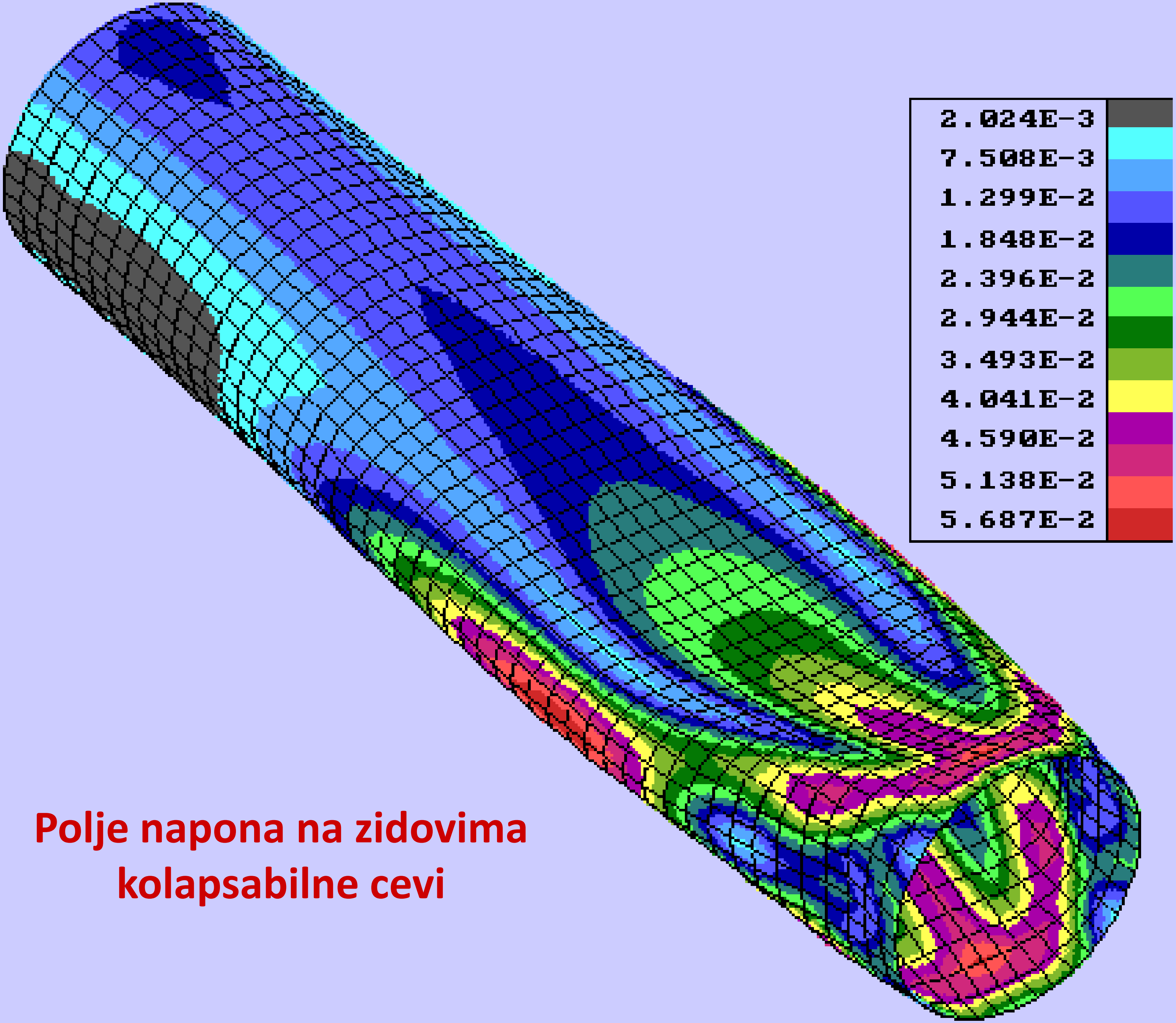
Deformisana cev na kraju procesa propadanja

Dijagram pritiska duž aksijalne ose kolapsibilne cevi

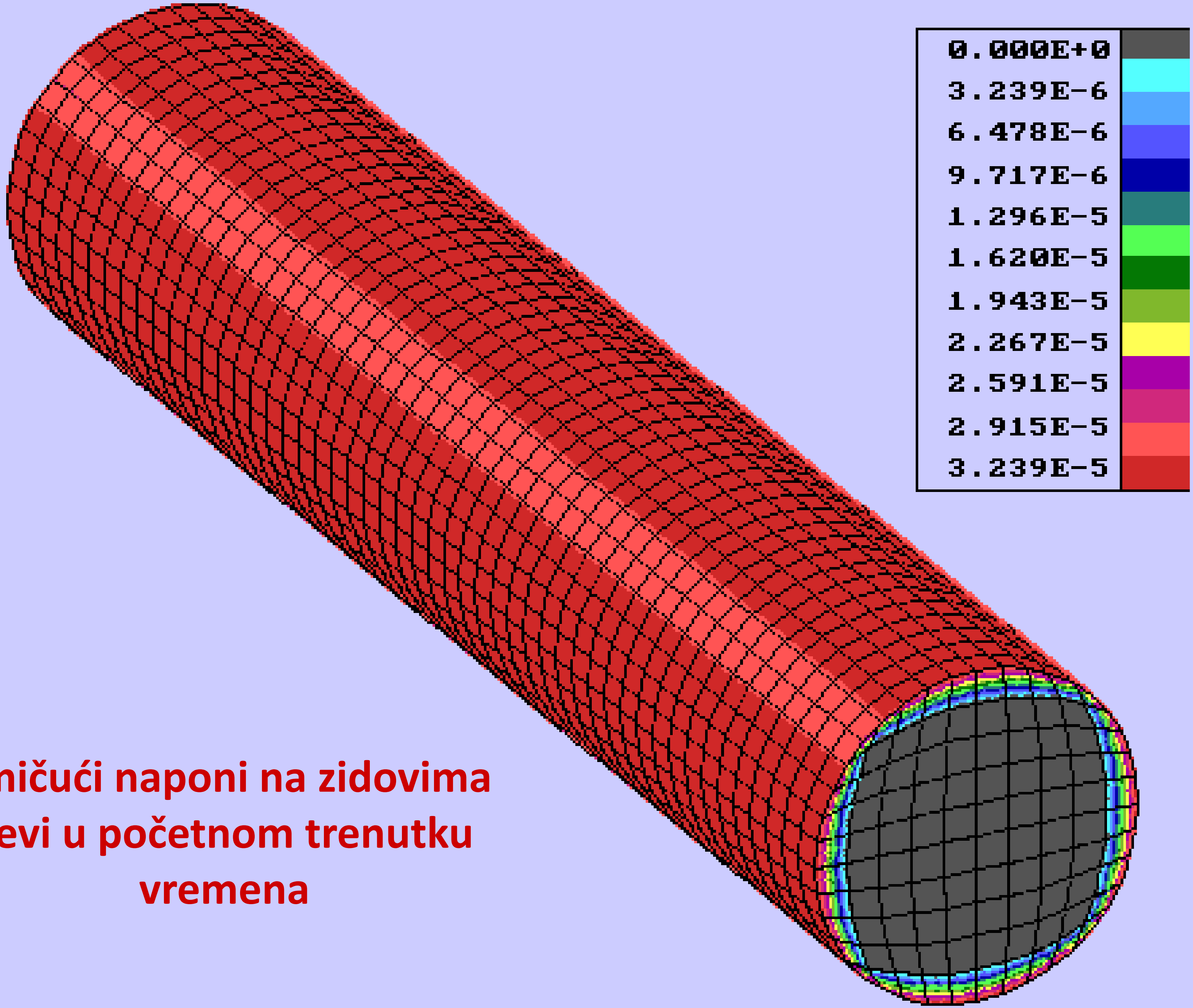


Polje pritisaka fluida u kolapsibilnoj cev

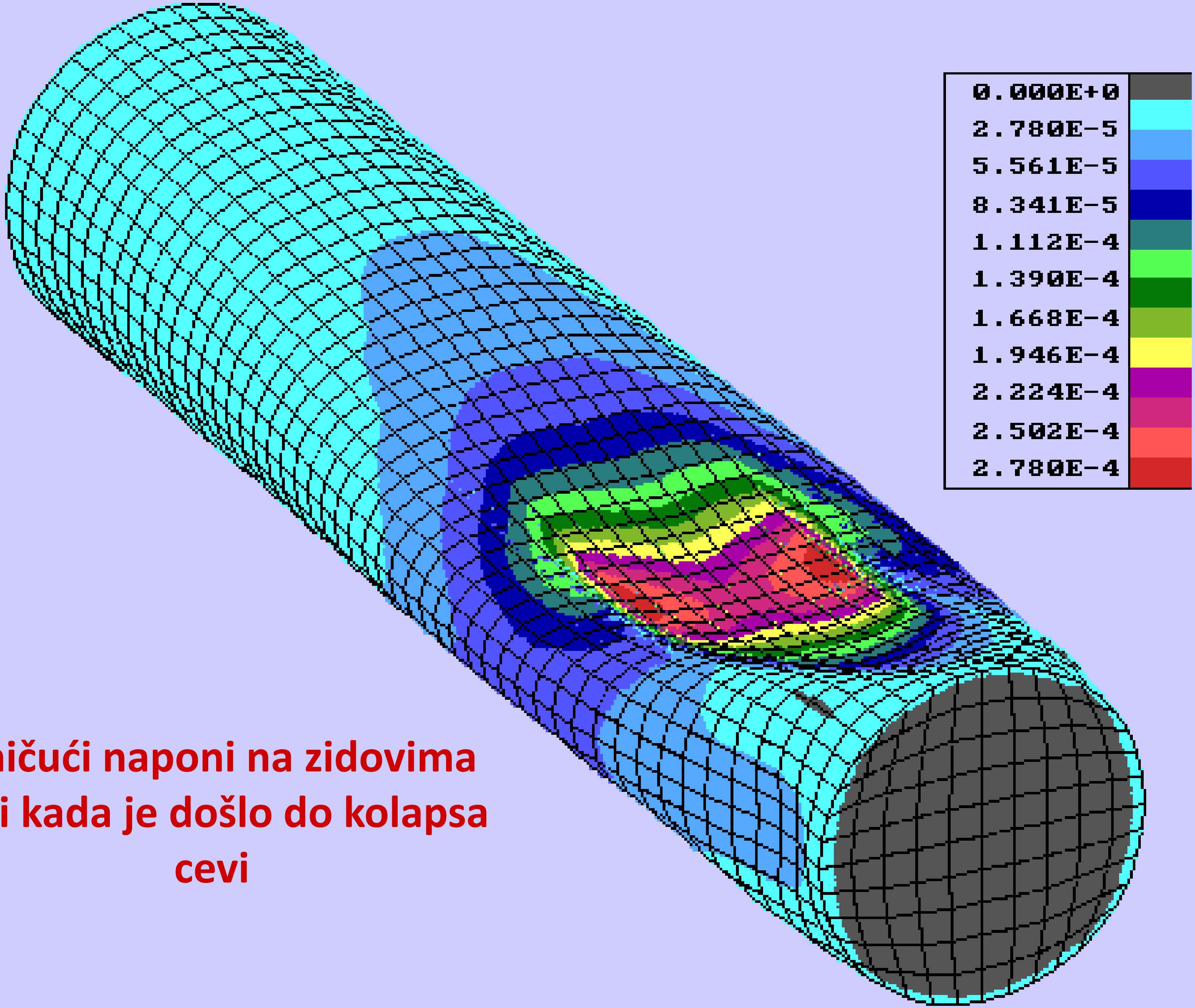




Polje napona na zidovima kolapsabilne cevi

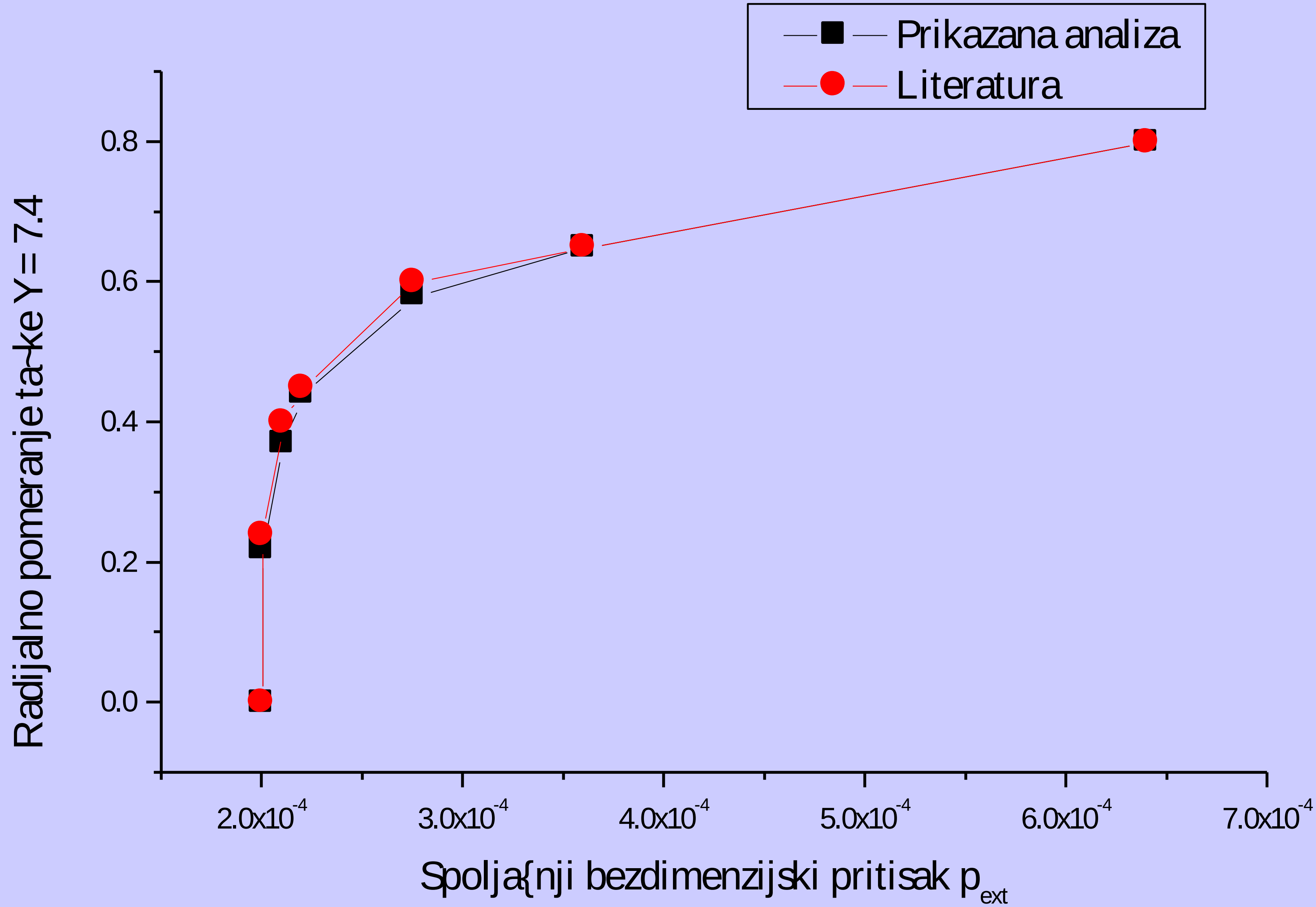


Smičući naponi na zidovima cevi u početnom trenutku vremena



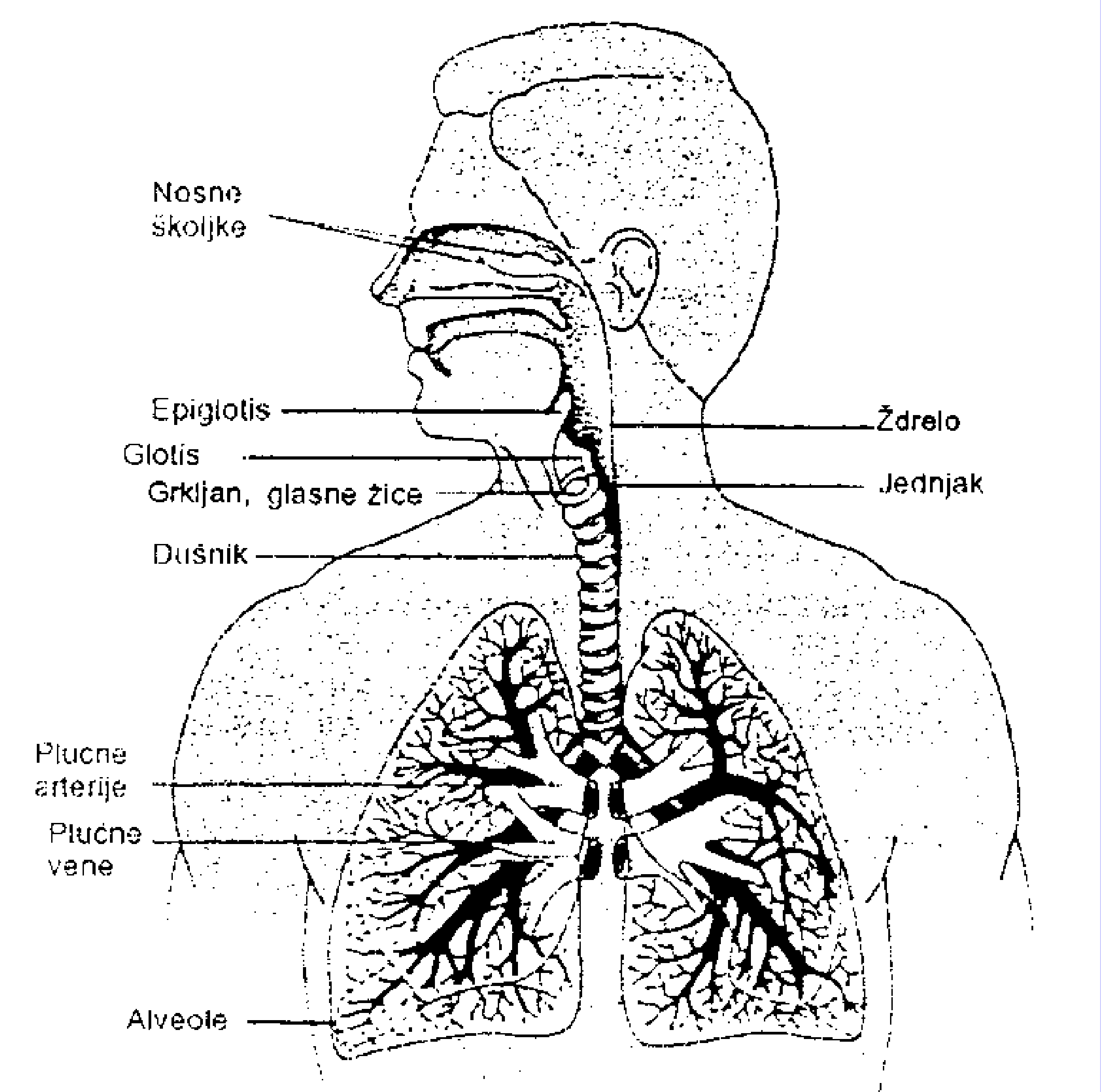
Smičući naponi na zidovima cevi kada je došlo do kolapsa cevi

Radijalno pomeranje tačke na bezdimenzijskom aksijalnom rastojanju Y=7.4 u funkciji spoljašnjeg pritiska p_{ext}

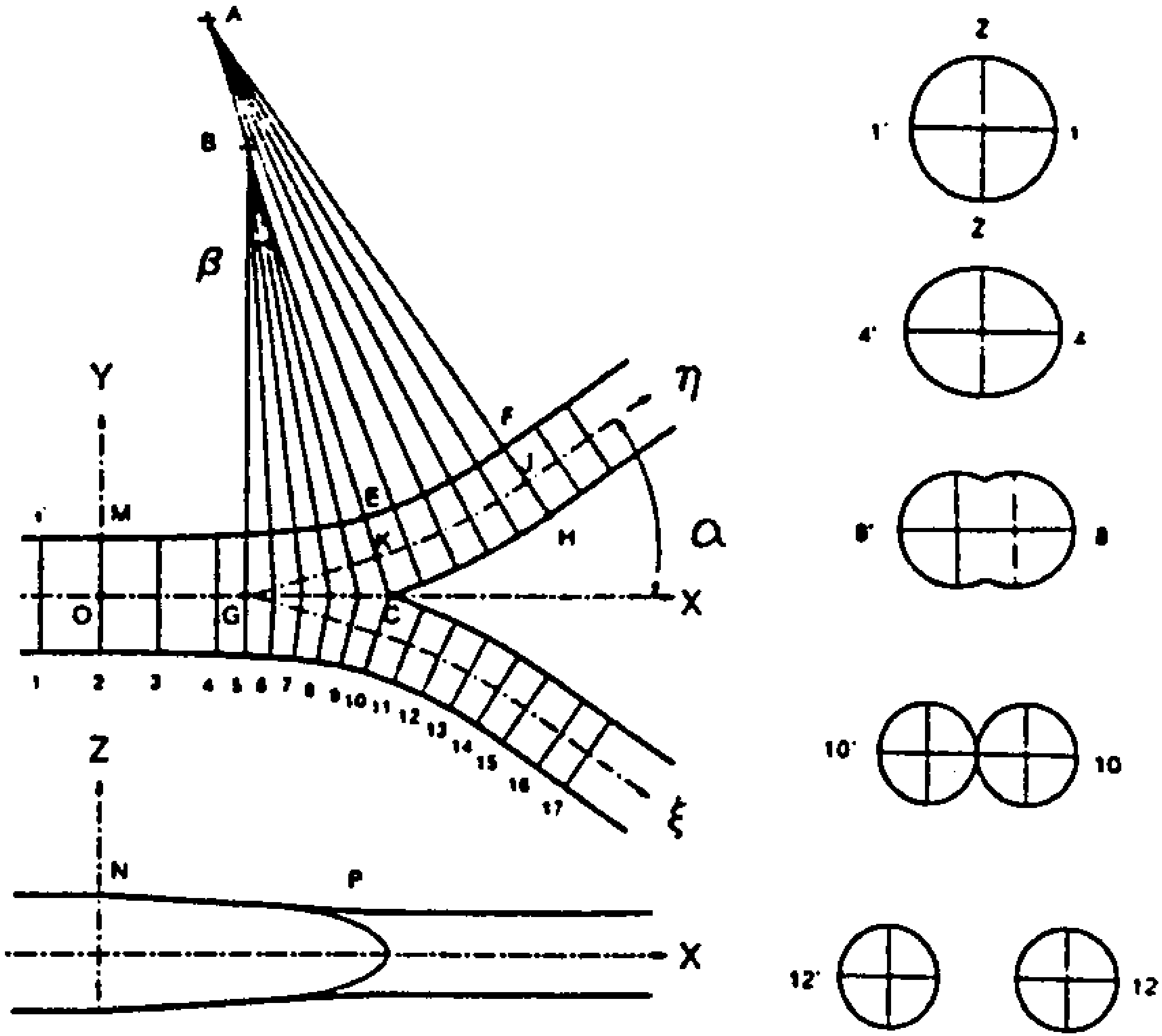


6. FIZIOLO[KA STRUJANJA U RESPIRATORNOM SISTEMU

Šematski prikaz respiratornog sistema

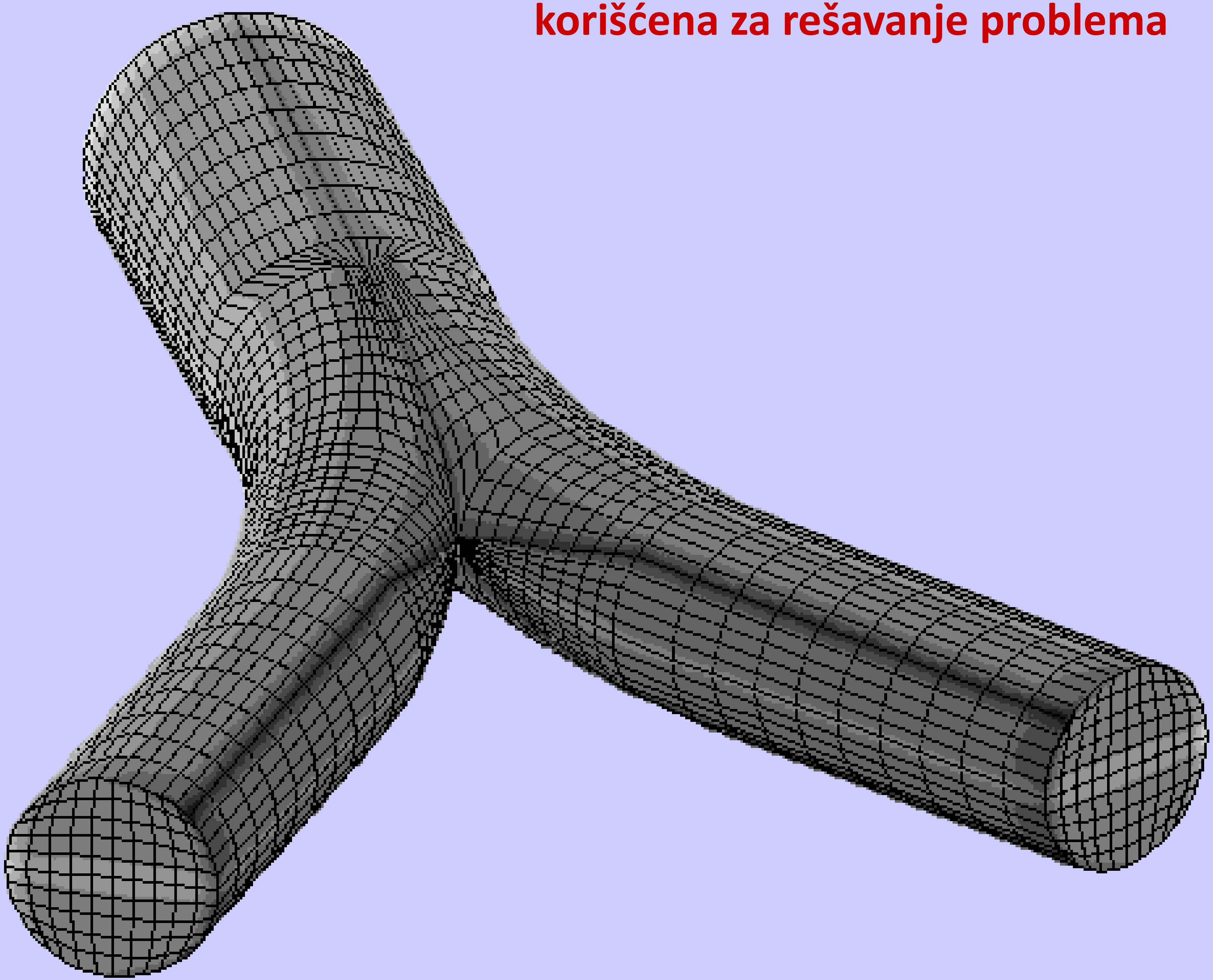


6.3.1 Inspirativno i ekspirativno strujanje u modelu bifurkacije respiratornog sistema

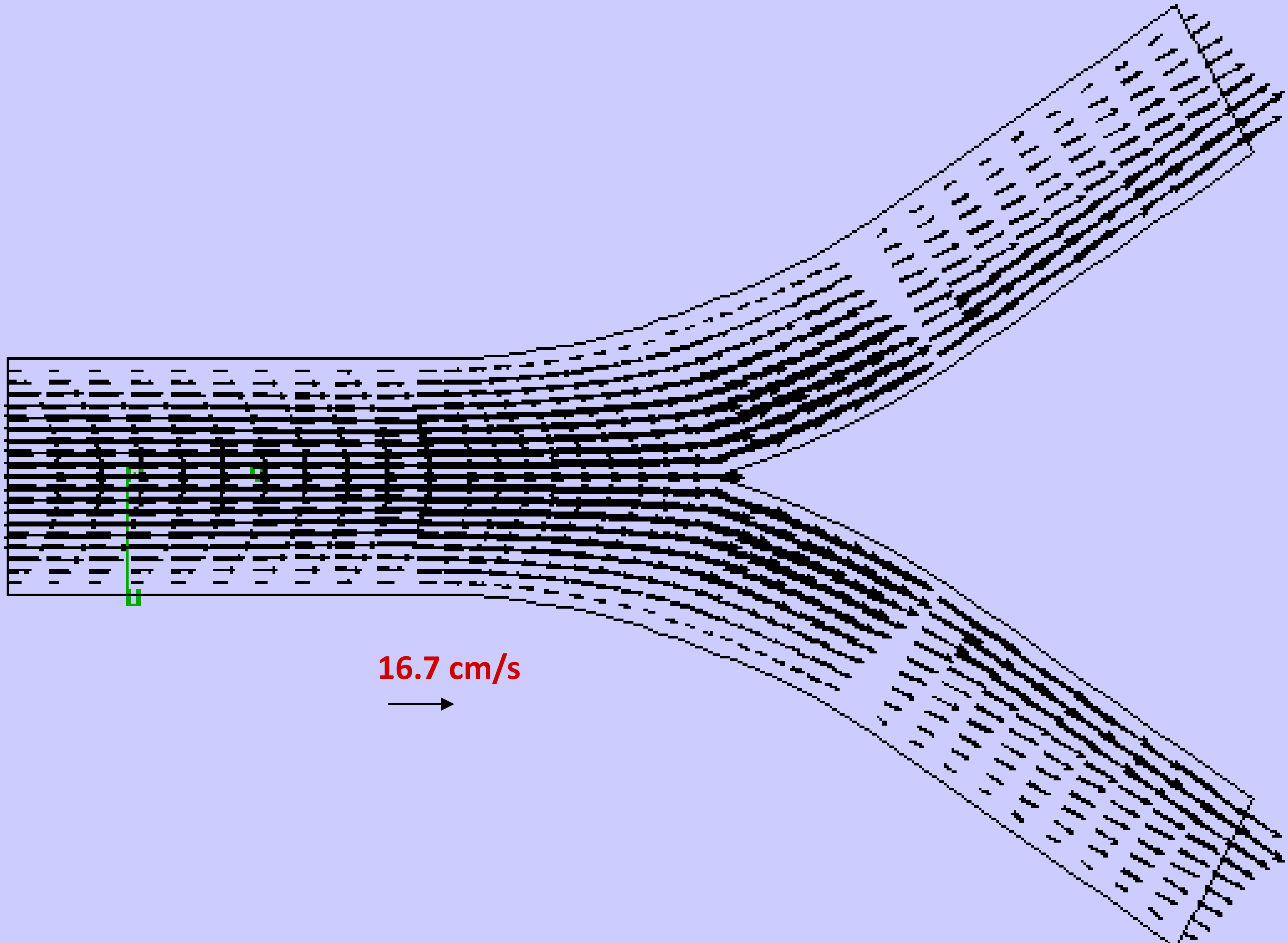


D=3.81 cm
L=2.5D
$2d^2/D^2=1$
R=7d
$\alpha=35°$
$\beta=18°$

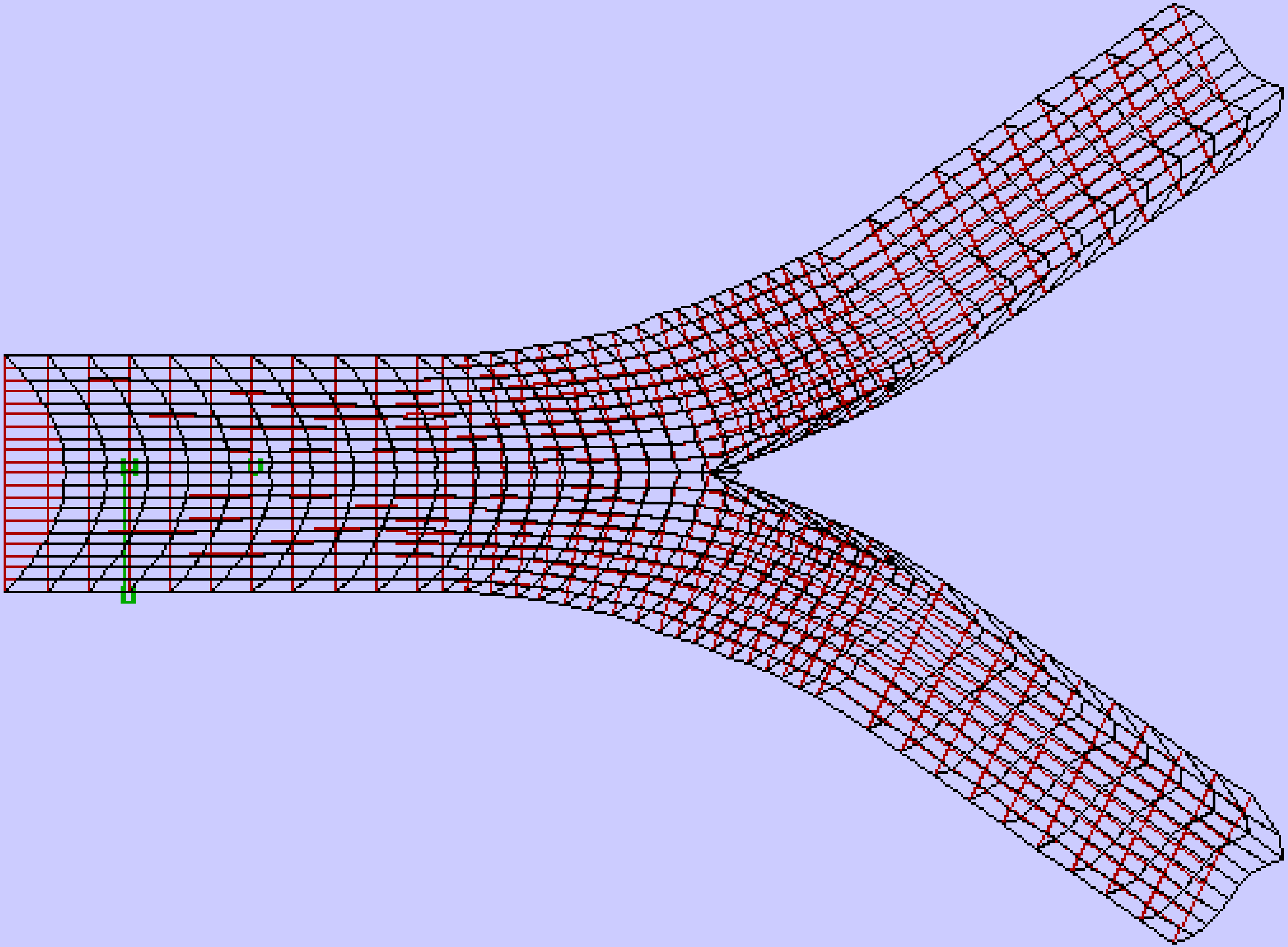
Mreža konačnih elemenata korišćena za rešavanje problema



6.3.1a Inspirativno strujanje

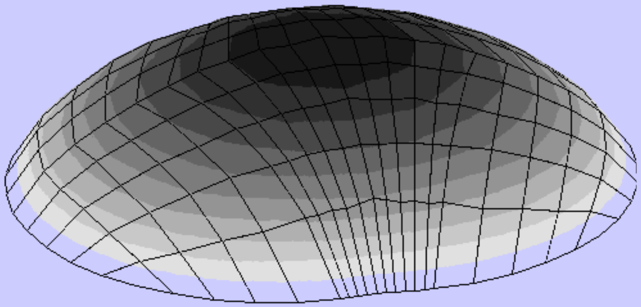


Vektorsko polje brzina u bifurkacionoj ravni za stacionarno inspiratorno strujanje skalirano u odnosu na maksimalnu brzinu od 16.7 cm/s

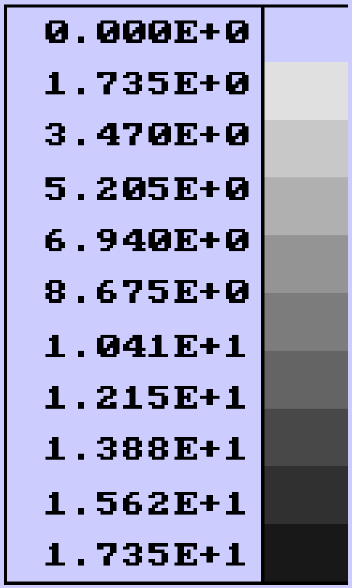


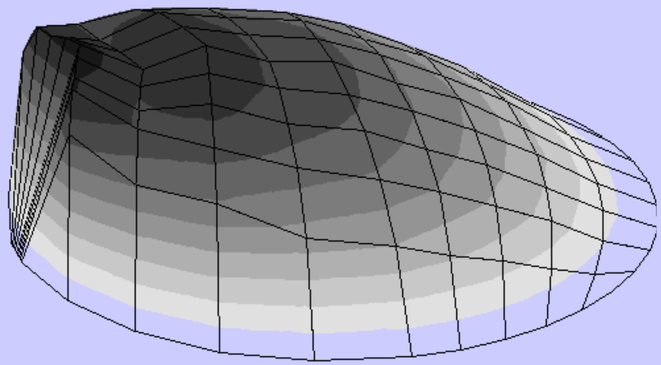
Profili aksijalne brzine u bifurkacionoj ravni za stacionarno inspiratorno strujanje

3-D prikaz aksijalne brzine u bifurkacionoj ravni za stacionarno inspiratorno strujanje

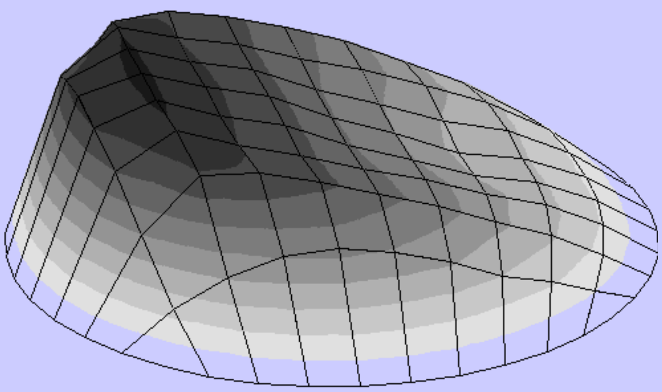


Presek 5



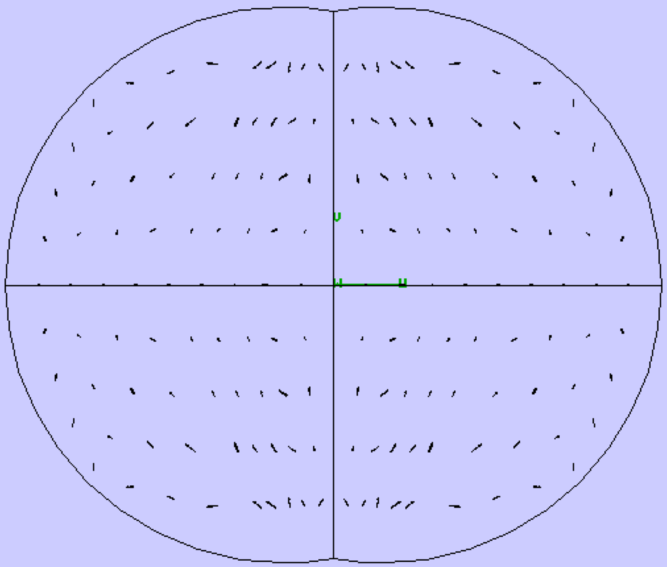


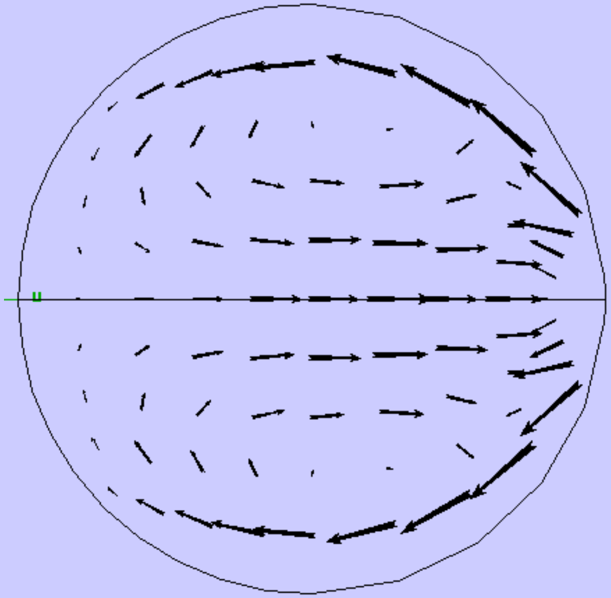
Presek 10

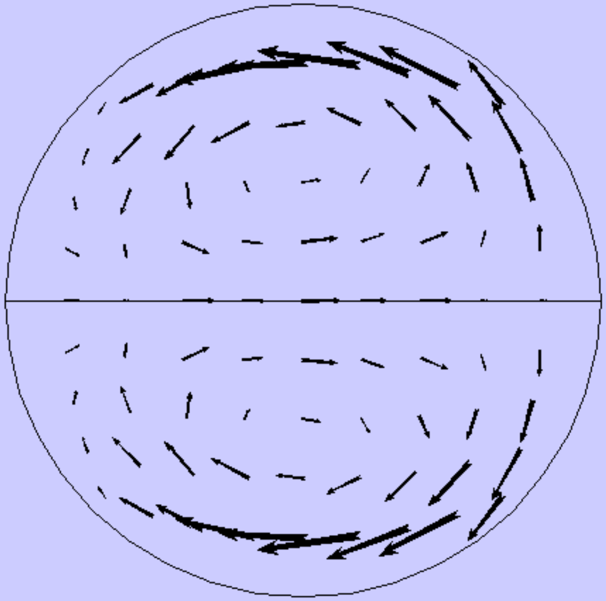


Presek 15

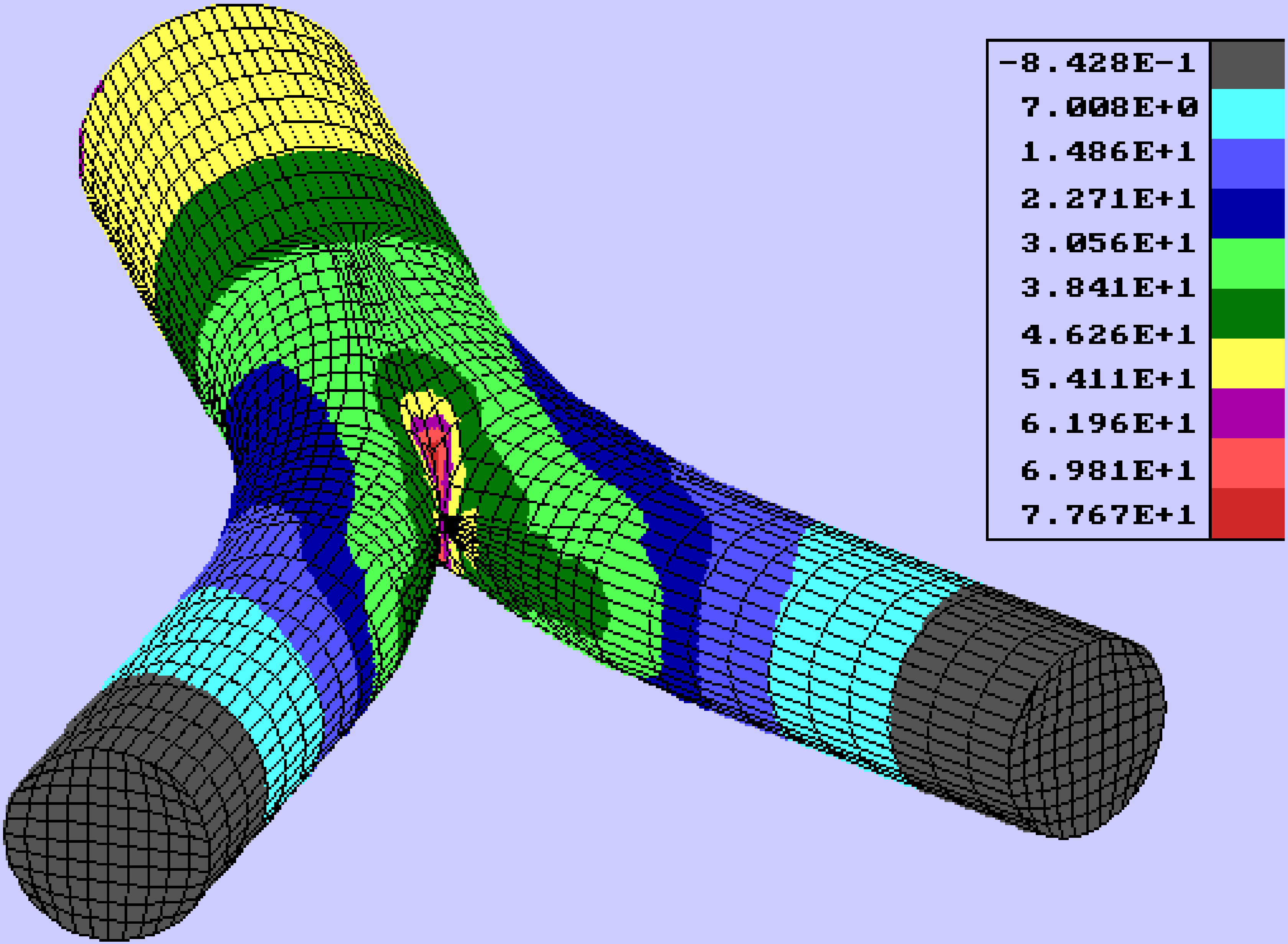
Vektorski prikaz polja radijalne brzine u bifurkacionoj ravni za stacionarno inspiratorno strujanje; skalirano u odnosu na maksimalnu brzinu 3.17 cm/s


Presek 5

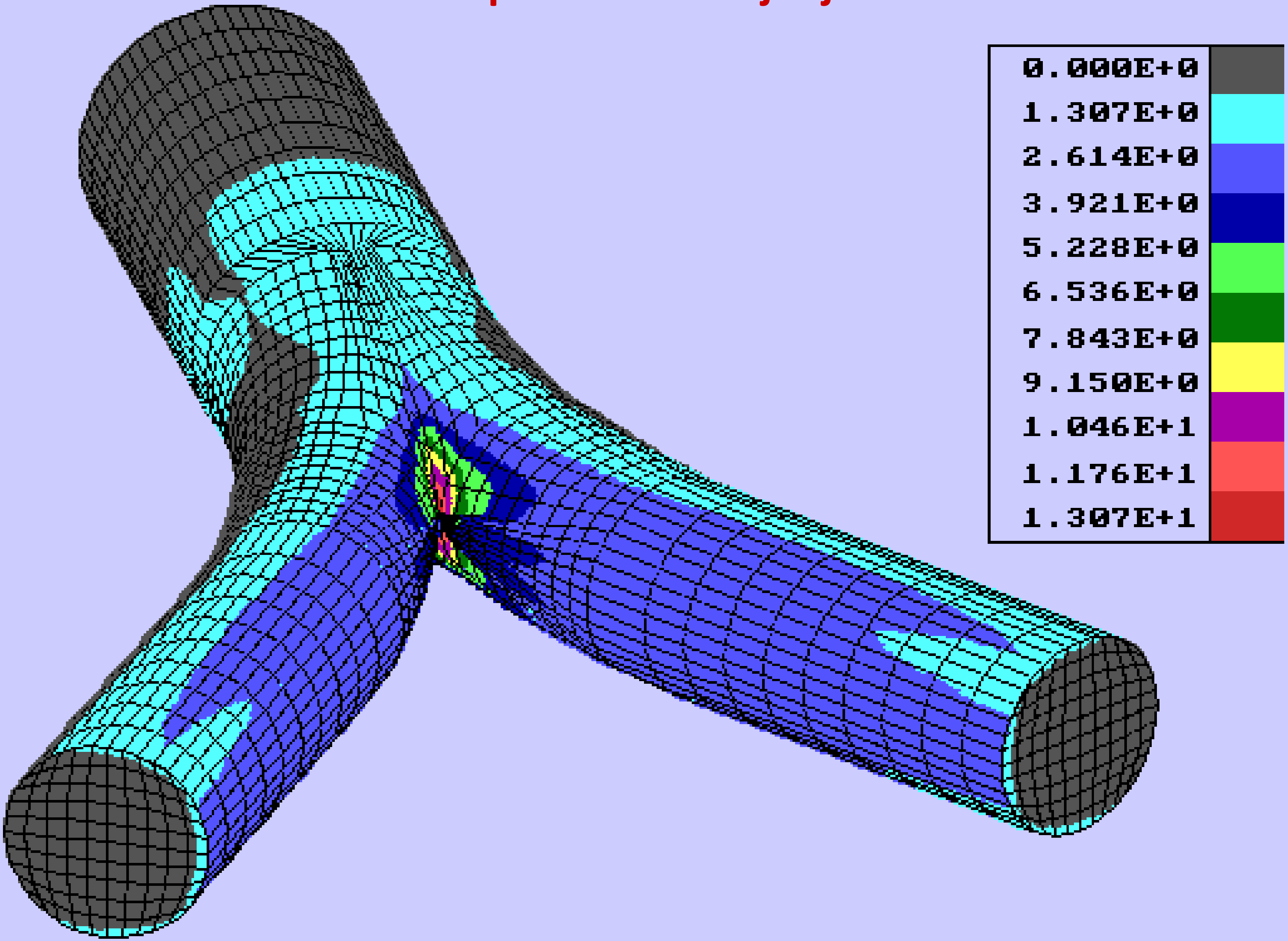

Presek 10


Presek 15

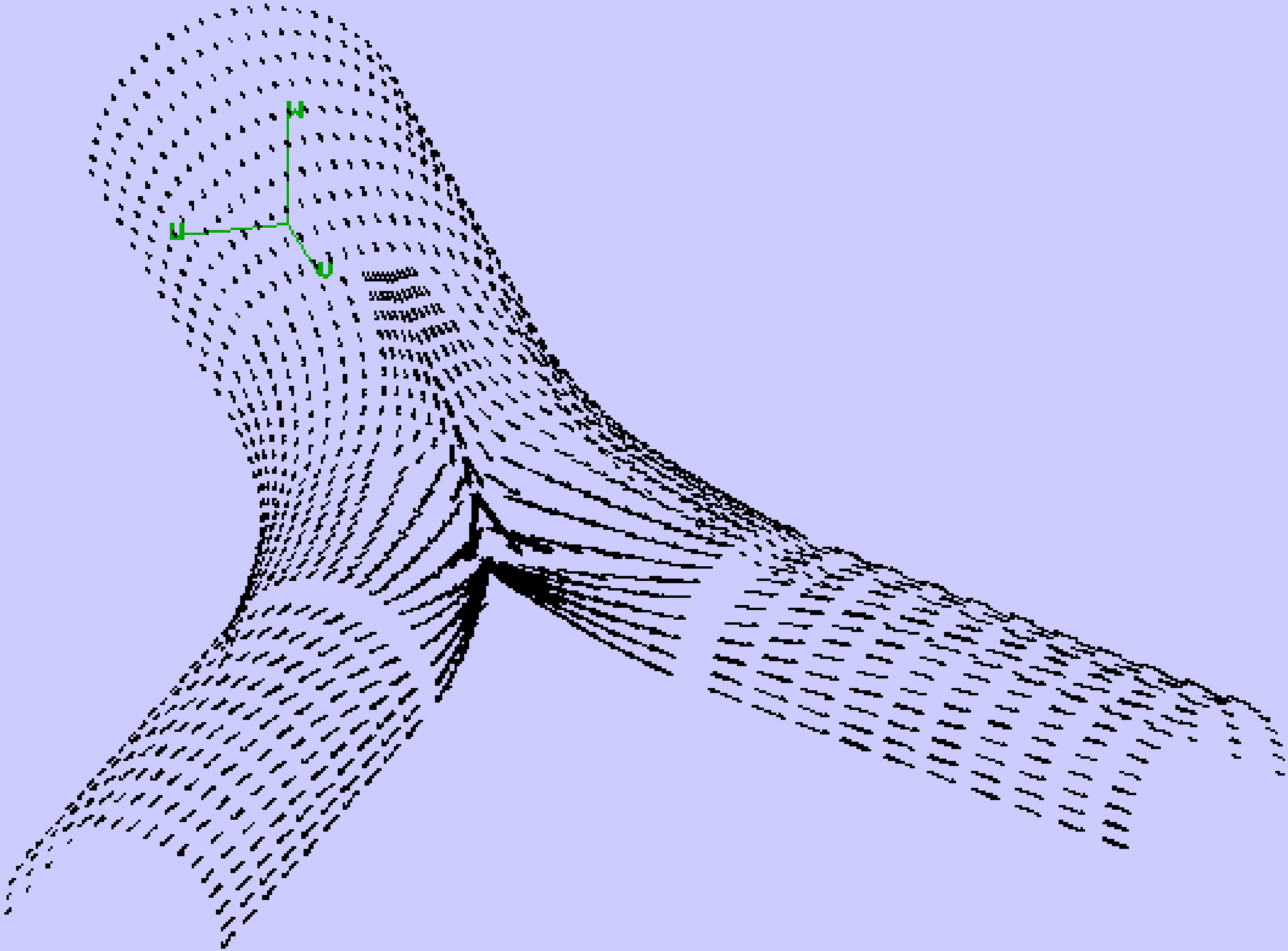
Polje pritisaka u 3-D modelu bifurkacije za stacionarno inspiratorno strujanje



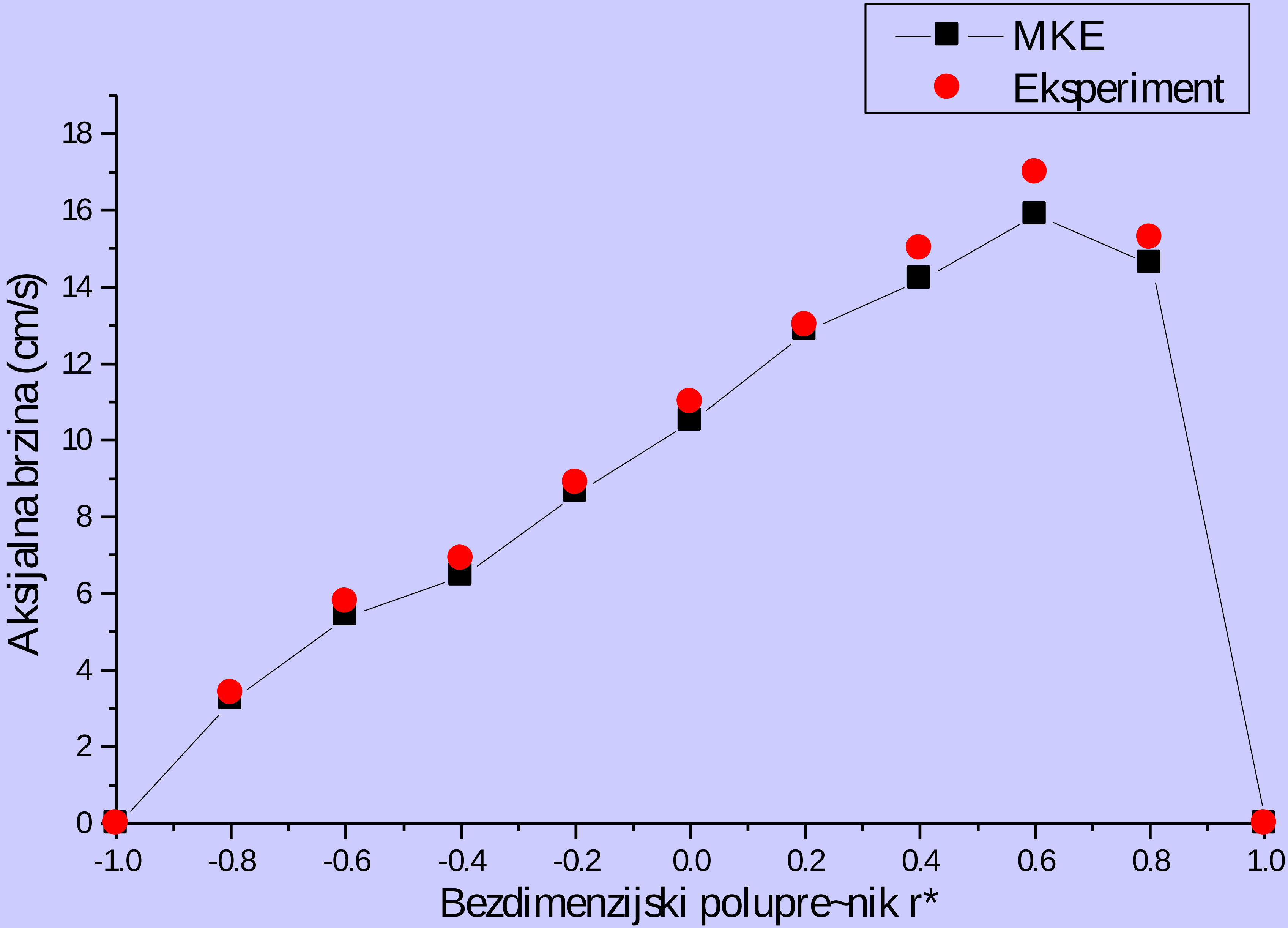
Polje smičućih napona u 3-D modelu bifurkacije za stacionarno inspiratorno strujanje



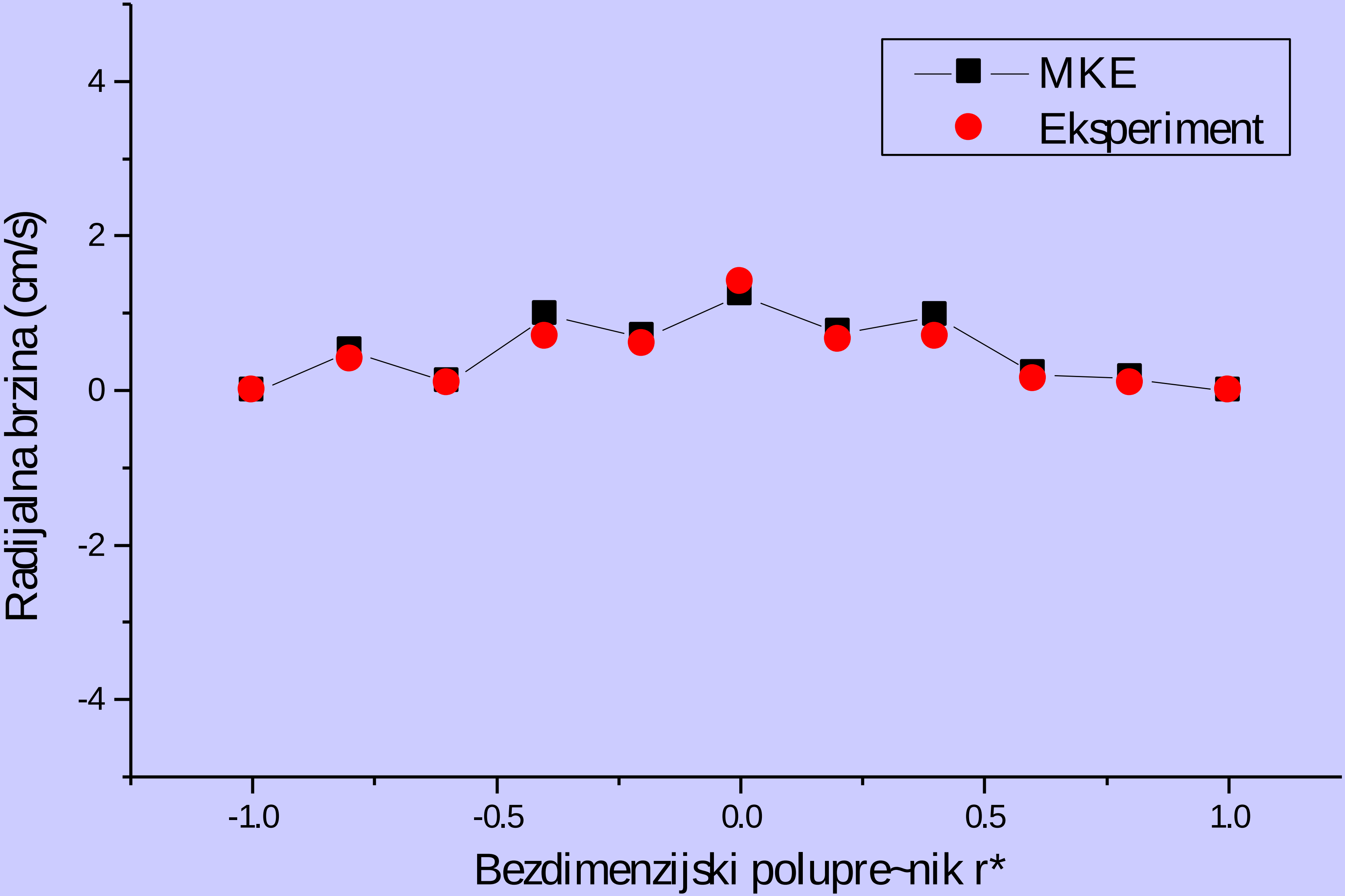
Vektorsko polje smičućih napona u 3-D modelu bifurkacije za stacionarno inspiratorno strujanje



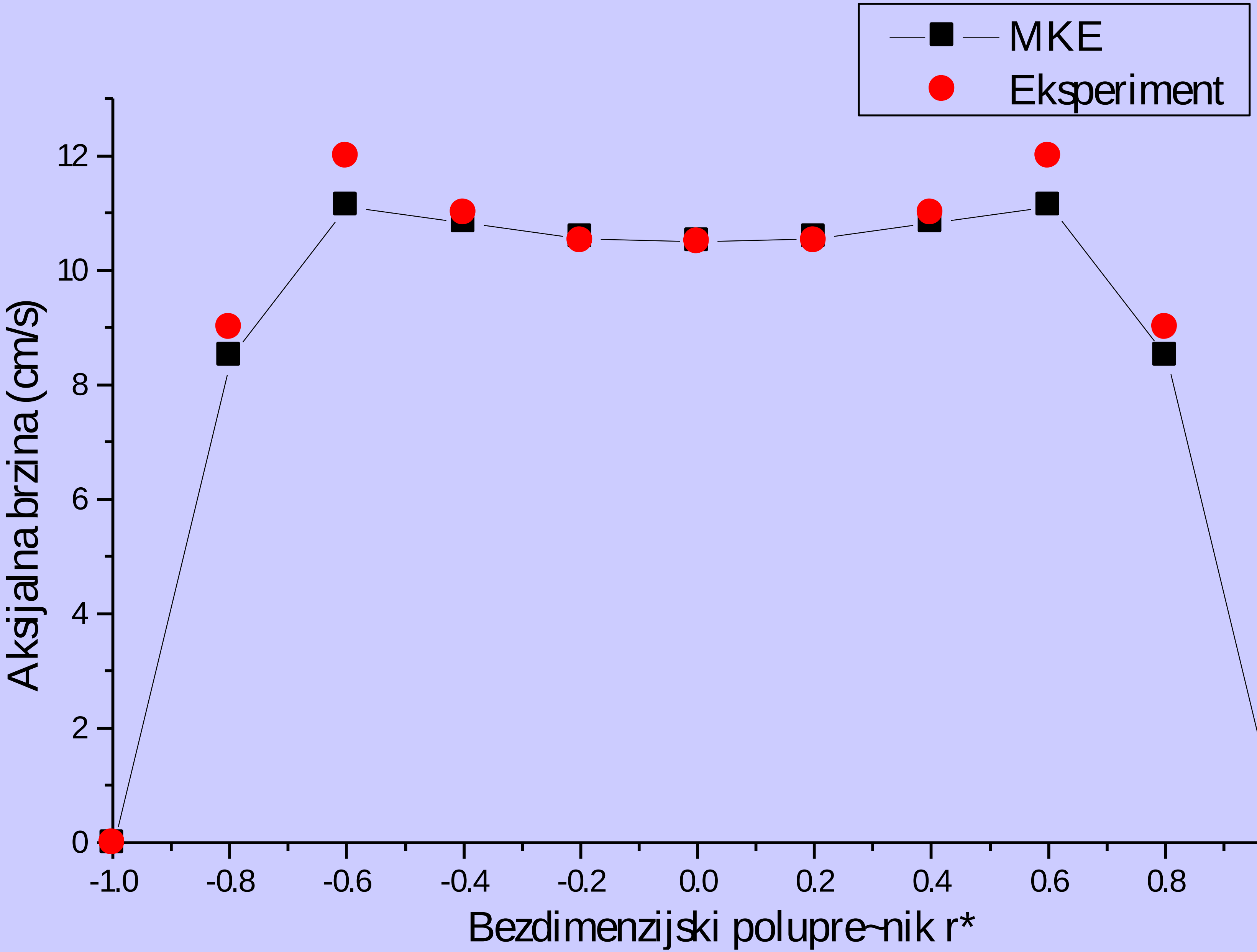
Numerički i eksperimentalni rezultati za aksijalnu brzinu u preseku 15 u bifurkacionoj ravni za stacionarno inspiratorno strujanje



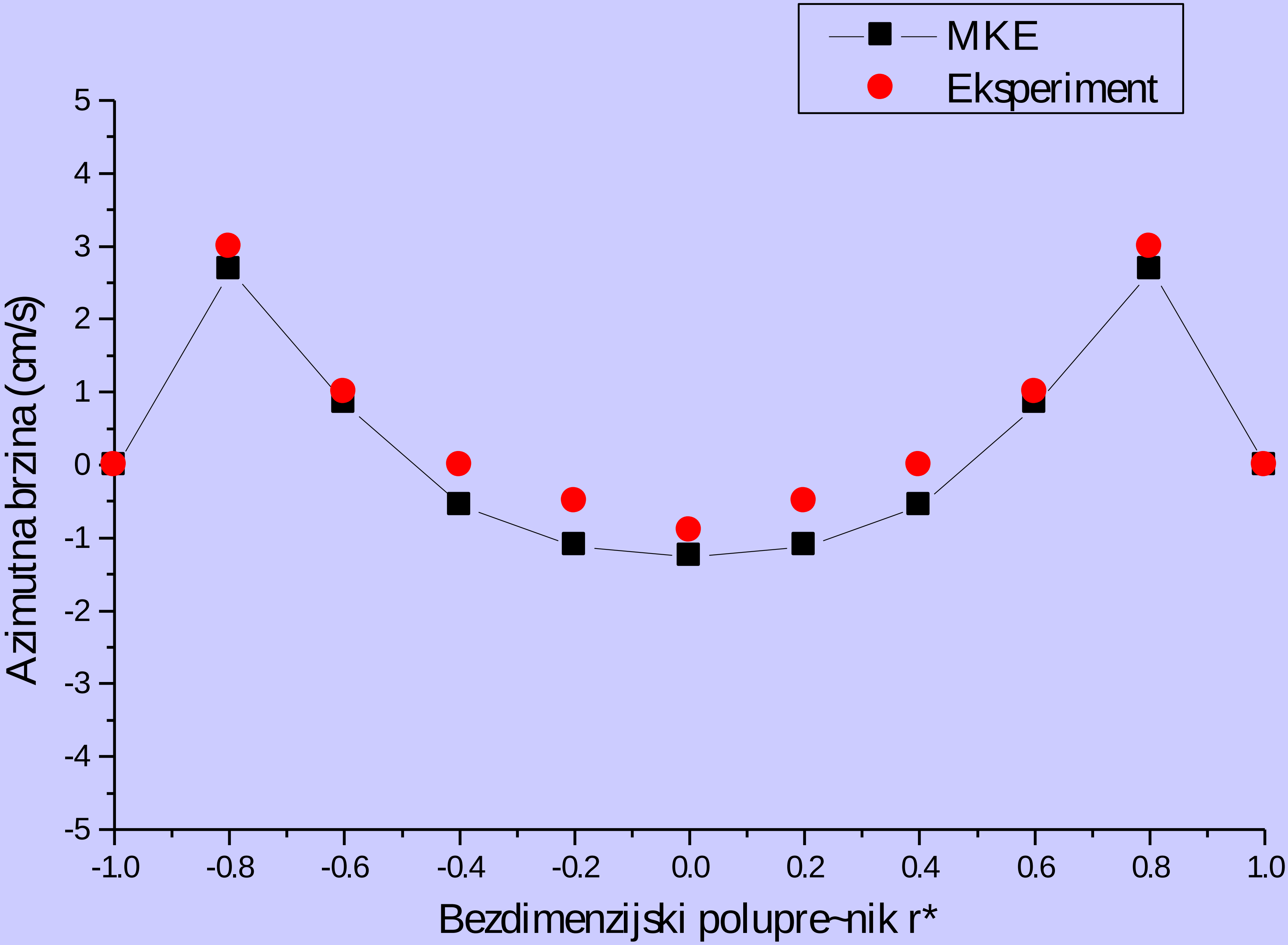
Numerički i eksperimentalni rezultati za radijalnu brzinu u preseku 15 u bifurkacionoj ravni za stacionarno inspiratorno strujanje



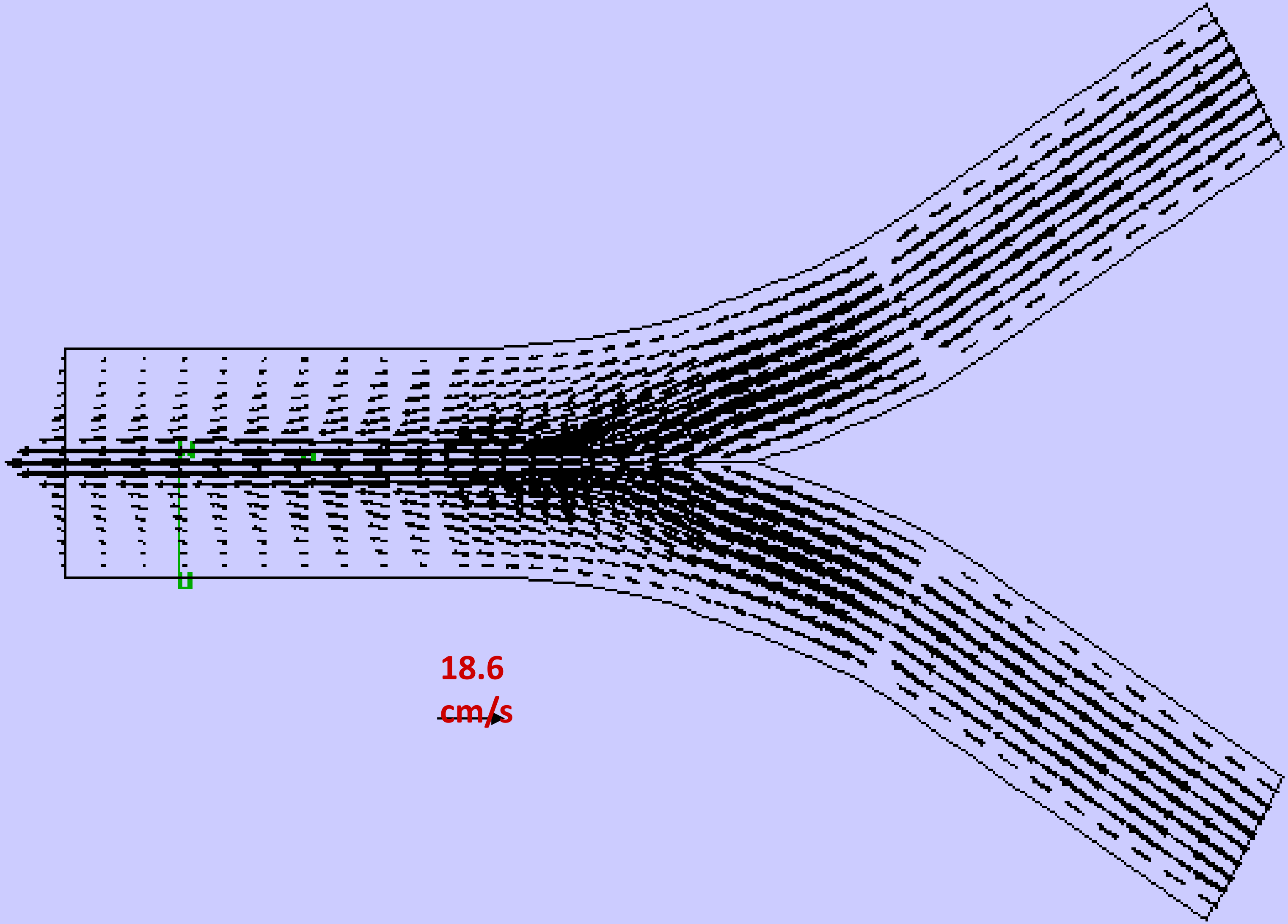
Numerički i eksperimentalni rezultati za radijalnu brzinu u preseku 15 u vertikalnoj ravni za stacionarno inspiratorno strujanje



Numeri~kih i eksperimentalni rezultati za azimutnu brzinu u preseku 15 u vertikalnoj ravni za stacionarno inspiratorno strujanje

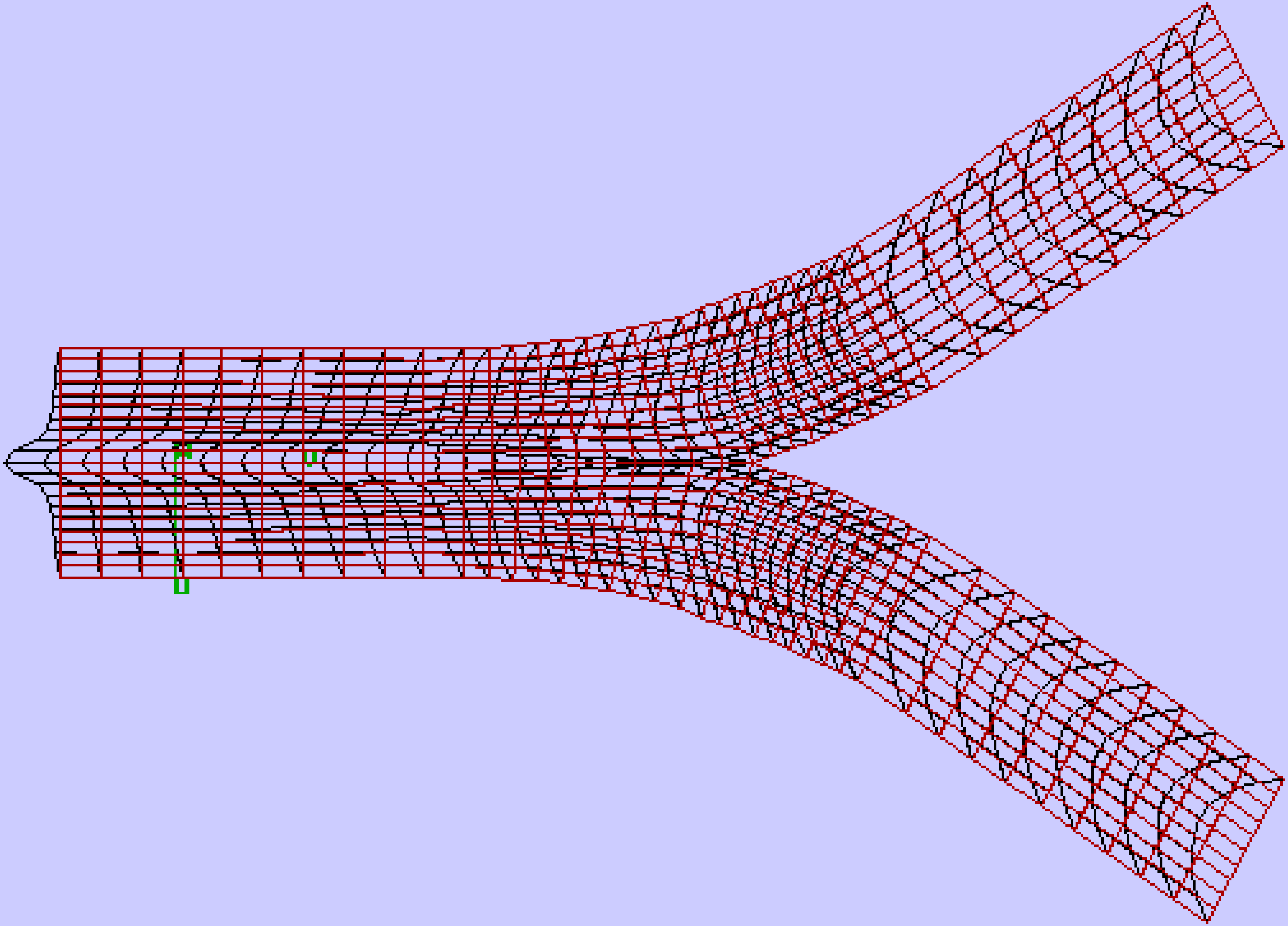


6.3.1.b Ekspiratorno strujanje

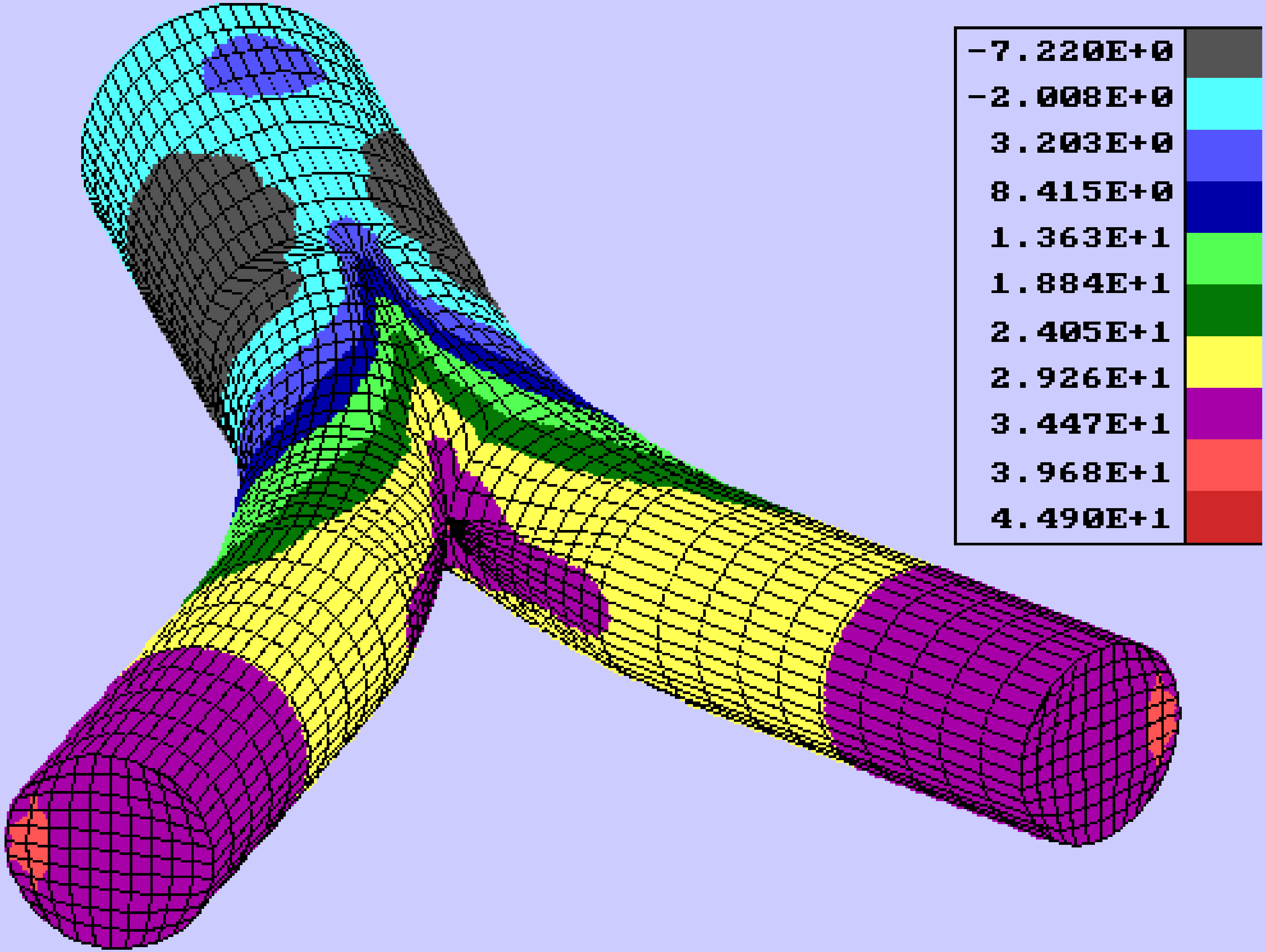


Vektorsko polje brzine u bifurkacionoj ravni za stacionarno ekspiratorno strujanje

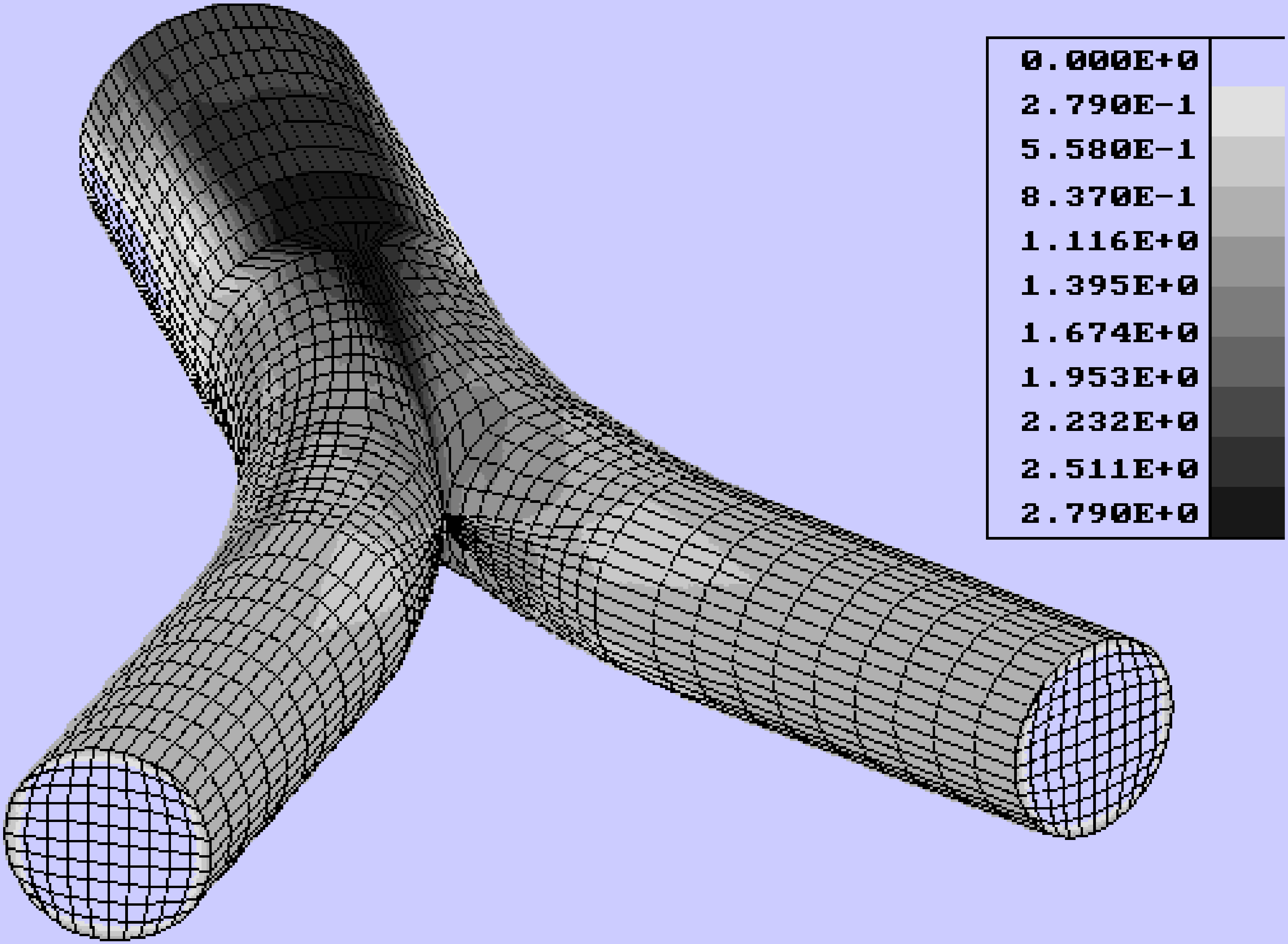
Profili polja brzine u bifurkacionoj ravni za stacionarno ekspiratorno strujanje



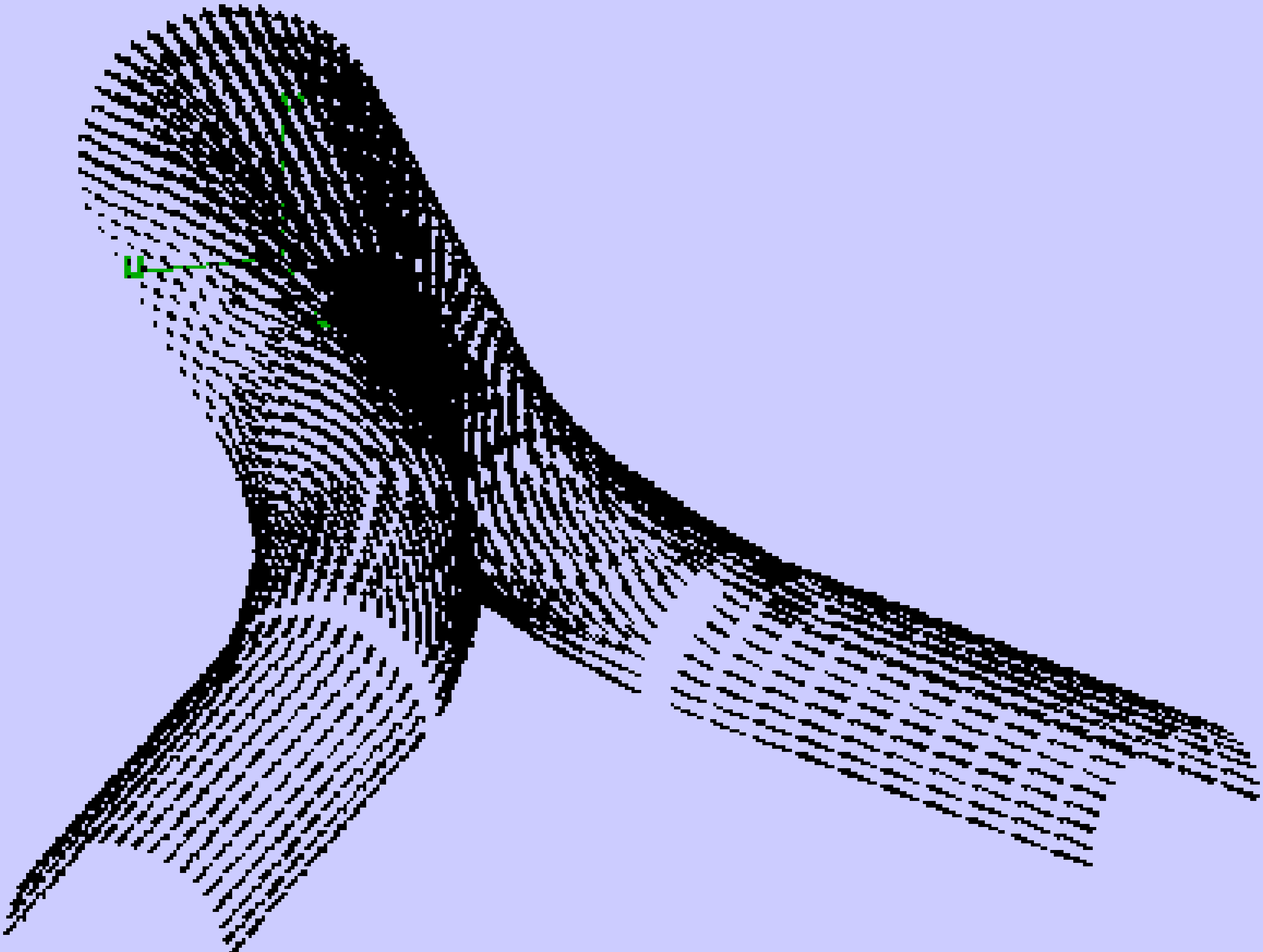
Polje pritiska u 3-D modelu bifurkacije za stacionarno ekspiratorno strujanje



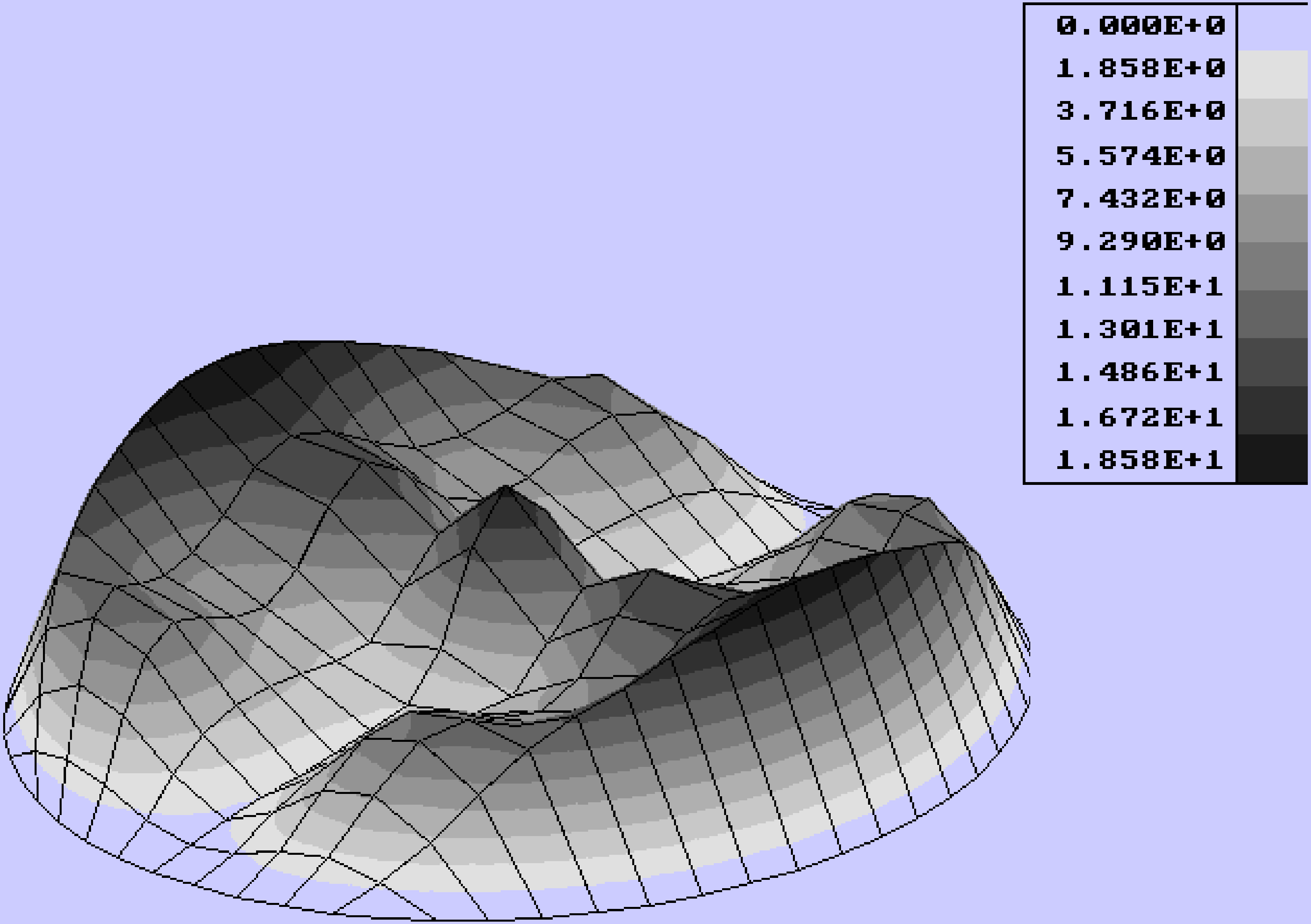
Polje smičućih napona na 3-D modelu bifurkacije za stacionarno ekspiratorno strujanje



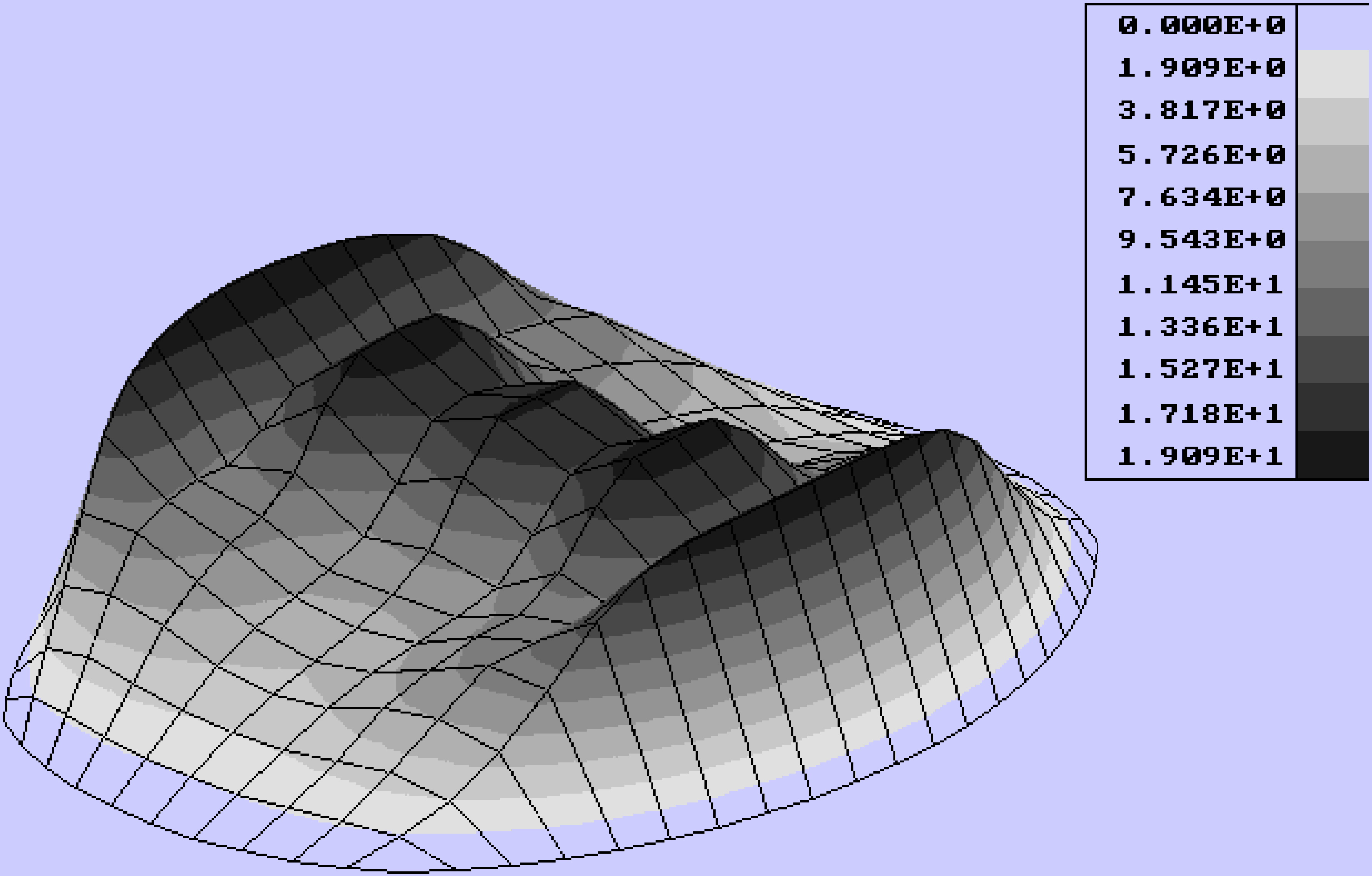
Vektorsko polje smičućih napona na 3-D modelu bifurkacije za stacionarno ekspiratorno strujanje



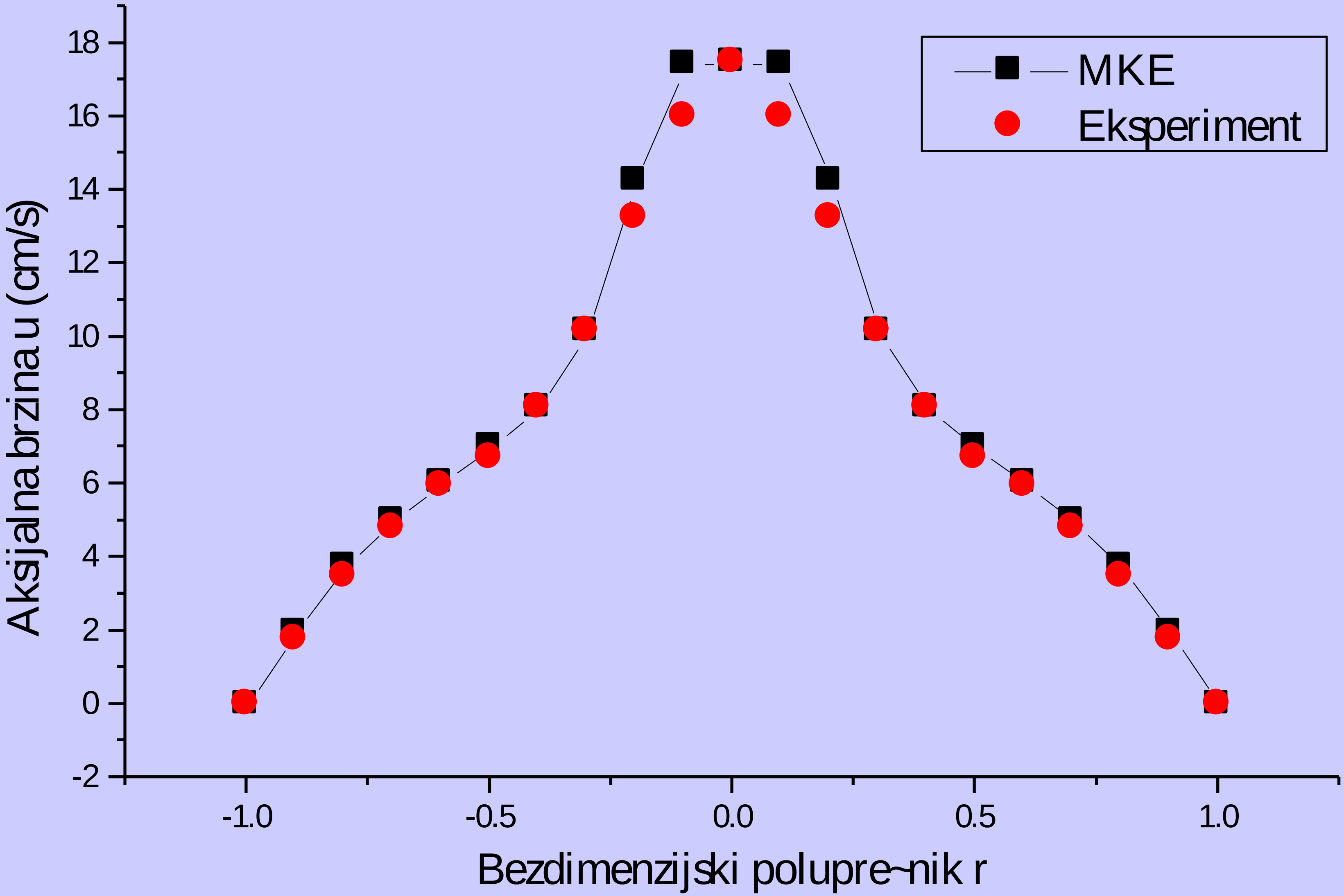
3-D prikaz aksijalne brzine za stacionarno ekspiratorno strujanje u preseku 1



3-D prikaz aksijalne brzine za stacionarno ekspiratorno strujanje u preseku 5



Numerički i eksperimentalni rezultati za aksijalnu brzinu u bifurkacionoj ravni u preseku 5, za stacionarno ekspiratorno strujanje



Numerički i eksperimentalni rezultati za radijalnu brzinu u bifurkacionoj ravni u preseku 5, za stacionarno ekspiratorno strujanje