MEHANIKA FLUIDA - KONTROLNI 8

STUDENT ______________________

1	2	3	Σ

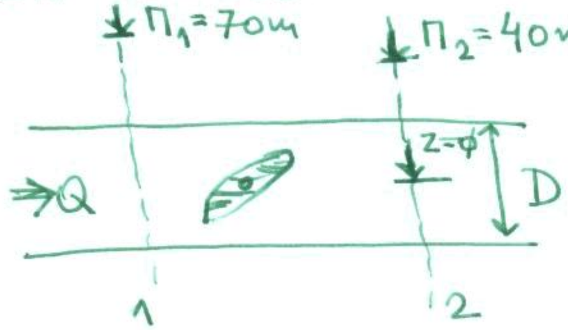
(1) DAT JE RASPORED BRZINA IZMEĐU DVE NEPOKRETNE PLOČE NA RASTOJANJU h

$$u_1 = u_0\left(1-\left(\frac{x_3-h/2}{h/2}\right)^2\right), \quad u_2=u_3=0 \qquad 0\le x_3\le h$$

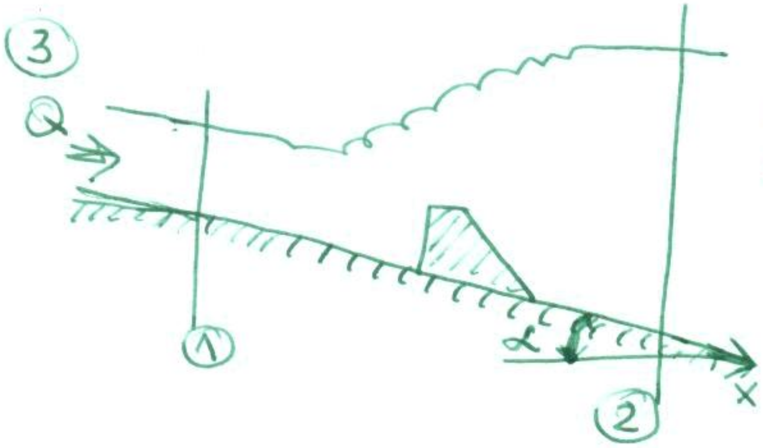
PRORAČUNOM SREDNJE BRZINE DOBIJEN JE ODNOS $V=\frac{2}{3}u_0$

KOLIKI JE BUSINESOV KOEFICIJENT $\beta=?$

(2) U CEVI SE NALAZI LEPTIRASTI ZATVARAČ. ODREDITI SILU NA ZATVARAČ AKO JE $D=0.3\,m$, $Q=60\,l/s$, $\rho=1000\,kg/m^3$.



(3)



NA SLICI JE NACRTAN HIDRAULIČKI SKOK U NAGNUTOM KANALU, SA "ZUBIMA" ZA POBOLJŠANJE USLOVA U FORMIRANJU SKOKA. NACRTATI NA SKICI SILE P_1, P_2, I_1, I_2, G_x, T, K U SVOJIM PRAVIM SMEROVIMA.

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(1) $$\beta = \frac{1}{Q\cdot V}\int_A u^2 dA = \frac{u_0^2\cdot B}{V^2\cdot B\cdot h}\int_0^h\left(1-\left(\frac{x_3-h/2}{h/2}\right)^2\right)^2 dx_3 = 1.20$$

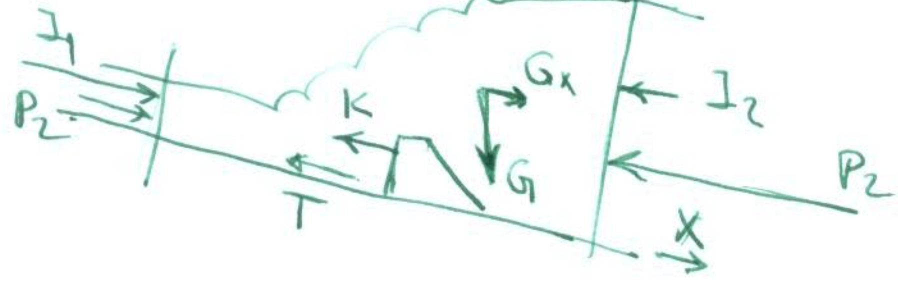
(VIDETI PRILOG)

(2)

$I_1+P_1 = F+I_2+P_2 \qquad I_1=I_2$

$F = P_1-P_2 = \rho g(\Pi_1-\Pi_2)\cdot A = \rho g(70-40)\cdot\frac{D^2\pi}{4} = 20.8\ kN$

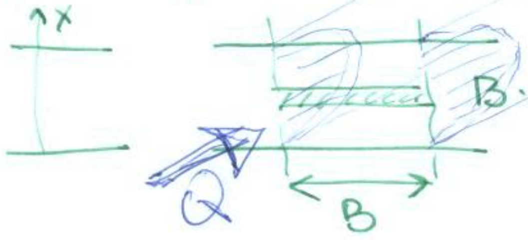
(3)



DAT JE RASPORED BRZINA IZMEĐU DVE NEPOKRETNE PLOČE

$$u_1 = u_0\left(1-\left(\frac{x_3-h/2}{h/2}\right)^2\right) \qquad 0 \le x_3 \le h \qquad u_2 = u_3 = \emptyset$$

NAĆI V i β.



$$V = \frac{1}{A}\int_A u\, dA =$$

$$V = \frac{u_0}{A}\int_0^h \left(1-\left(\frac{x_3-h/2}{h/2}\right)^2\right) B \cdot dx_3 =$$

$$V = \frac{B \cdot u_0}{B \cdot h}\left[\int_0^h dx_3 - \frac{4}{h^2}\int_0^h \left(x_3-\frac{h}{2}\right)^2 dx_3\right]$$

$$V = \frac{u_0}{h}\left[h - \frac{4}{h^2}\int_0^h \left(x_3^2 - h\cdot x_3 + \frac{h^2}{4}\right)dx_3\right]$$

$$V = \frac{u_0}{h}\left[h - \frac{4}{h^2}\left(\frac{h^3}{3} - \frac{h^3}{2} + \frac{h^3}{4}\right)\right]$$

$$V = \frac{u_0}{h}\left[h - \frac{4}{3}h + 2h - h\right] = \frac{2}{3}\frac{u_0 h}{h} = \frac{2}{3}u_0 \;//$$

$$\beta = \frac{1}{Q \cdot V}\int_A u^2 dA = \frac{u_0^2 \cdot B}{V^2 \cdot A}\int_0^h \left(1-\left(\frac{x_3-h/2}{h/2}\right)^2\right)^2 dx_3$$

$$\beta = \frac{u_0^2 B}{V^2 \cdot A}\left[\underbrace{\int_0^h dx_3}_{I1} - \underbrace{\int_0^h 2\left(\frac{x_3-h/2}{h/2}\right)^2 dx_3}_{I2} + \underbrace{\int_0^h \left(\frac{x_3-h/2}{h/2}\right)^4 dx_3}_{I3}\right]$$

$$I1 = \int_0^h dx_3 = +h$$

$$I2 = -\int_0^h 2\left(\frac{x_3 - h/2}{h/2}\right)^2 dx_3 = -\frac{8}{h^2}\int_0^h \left(x_3^2 - h x_3 + \frac{h^2}{4}\right) dx =$$

$$= -\frac{8}{h^2}\left(\frac{h^3}{3} - \frac{h^3}{2} + \frac{h^3}{4}\right) = -\frac{\cancel{8}^{\,2}}{h^2}\,\frac{\overbrace{4h^3 - 6h^3 + 3h^3}^{1}}{\cancel{12}_{\,3}}$$

$$= -\frac{2}{3}h$$

$$I3 = \int_0^h \left(\frac{x_3 - \frac{h}{2}}{\frac{h}{2}}\right)^4 dx_3 = \frac{16}{h^4}\int_0^h \left(x_3^2 - h x_3 + \frac{h^2}{4}\right)^2 dx =$$

$$= \frac{16}{h^4}\int_0^h \left(x_3^4 - h x_3^3 + \frac{h^2}{4}x_3^2 - h x_3^3 + h x_3^3 - \frac{h^3}{4}x_3 + \frac{h^2}{4}x_3^2 - \frac{h^3}{4}x_3 + \frac{h^4}{16}\right) dx =$$

$$= \frac{16}{h^4}\int_0^h \left(x_3^4 - 2h x_3^3 + \frac{3}{2}h^2 x_3^2 - \frac{h^3}{2}x_3 + \frac{h^4}{16}\right) dx =$$

$$= \frac{16}{h^4}\left(\frac{h^5}{5} - \frac{h^5}{2} + \frac{h^5}{2} - \frac{h^5}{4} + \frac{h^5}{16}\right) = \frac{16}{h^4}\cdot\frac{h^5}{5\cdot 16} = \frac{1}{5}h$$

$$\beta = \frac{u_0^2 B}{V^2 \cdot h \cdot B}\left(h - \frac{2}{3}h + \frac{1}{5}h\right) = \frac{8}{15}\,\frac{u_0^2 B\cdot h}{V^2 B\cdot h} = \frac{8}{15}\left(\frac{u_0}{V}\right)^2$$

Iz $V = \frac{2}{3}u_0 \Rightarrow \frac{u_0}{V} = \frac{3}{2}$

$$\beta = \frac{8}{15}\left(\frac{3}{2}\right)^2 = \frac{8\cdot 9}{15\cdot 4} = \frac{18}{15} = 1.20$$

RESAVANJE SA SMENOM (BUDO ZINDOVIĆ)

$$\beta = \frac{u_0^2}{h \cdot v^2} \underbrace{\int_0^h \left(1 - \left(\frac{x_3 - h/2}{h/2}\right)^2\right)^2 dx_3}$$

$$\frac{x_3 - \frac{h}{2}}{\frac{h}{2}} = t \qquad x_3 = \frac{h}{2}(t+1) \qquad dx_3 = \frac{h}{2}dt \qquad \int_0^h \rightarrow \int_{-1}^1$$

$$\Rightarrow \int_0^h \left(1 - \left(\frac{x_3 - h/2}{h/2}\right)^2\right)^2 dx_3 = \int_{-1}^1 (1-t^2)^2 \frac{h}{2} dt =$$

$$= \frac{h}{2}\int_{-1}^1 (1 - 2t^2 + t^4)\,dt = \frac{h}{2}\left[t - \frac{2}{3}t^3 + \frac{t^5}{5}\right]\Big|_{-1}^1 =$$

$$= \frac{h}{2} \cdot 2\left(1 - \frac{2}{3} + \frac{1}{5}\right) = \frac{8}{15}h$$

$$\beta = \frac{u_0^2 \cdot \frac{8}{15}h}{h \cdot \left(\frac{2}{3}u_0\right)^2} = \frac{\frac{8}{15}hu_0^2}{\frac{4}{9}hu_0^2} = \frac{8 \cdot 9}{15 \cdot 4} = \frac{6}{5} = 1{,}20$$