Making uncertainty analysis simple

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Introduction

• Uncertainty analysis is an important issue in urban drainage modelling

• Everybody agrees on that....
Knowledge on uncertainty is expanding...

- A lot of research in the past decade → many articles on that
- Main focus of Int. Working Group on Data&Models in the last 5 years

Now we know a lot about uncertainty!!

Typical researcher working with urban drainage models
...but there are still some issues

To create your uncertainty bounds, you can apply a threshold on likelihood of 0.124567

Threshold of 0.124567??????

All methods involve a degree of subjectivity

Assumptions difficult to express in tangible terms

Typical practitioner working daily with urban drainage models
...but there are still some issues

My model matches only 60% of the observations...

Well, if you knew how those data were taken... 60% is already a miracle..

Measurement uncertainty is seldom taken into consideration
Aim & Objectives

• To make uncertainty analysis simple and understandable
  • Try to reduce the subjectivity of the choices done when running uncertainty analysis
  • Describe the subjective choices in a more tangible manner
  • Introduce a criterion to assess model performance by considering measurement uncertainty
    • Vezzaro-McCarthy Criterion...VMC
Step 1

1. Define uncertainty intervals for each observed datapoint

Example of Rainfall Runoff model – predicting flows
Step 2

1. Define uncertainty intervals for each observed datapoint

2. Generate $N$ parameter sets

$\Theta_{1,2,...,N}$
Step 3

1. Define uncertainty intervals for each observed datapoint

2. Generate $N$ parameter sets

3. Run model for $N$ parameter sets and rank them

- $E=0.25$, $k=3$ (3rd)
- $E=0.30$, $k=2$ (2nd)
- $E=0.50$, $k=1$ (1st)
Step 4

1. Define uncertainty intervals for each observed datapoint
2. Generate $N$ parameter sets
3. Run model for $N$ parameter sets and rank them

Estimate model prediction bounds (e.g. $K=3$)

$K =$ number of included ranked simulations in the estimation of the model prediction bounds
Step 5

1. Define uncertainty intervals for each observed datapoint

2. Generate $N$ parameter sets

3. Run model for $N$ parameter sets and rank them

4. Estimate model prediction bounds (e.g. $K=3$)

5. Estimate intersection $\lambda$ (e.g. for $K=3$, $\lambda=66\%$)
Step 6

1. Define uncertainty intervals for each observed datapoint.
2. Generate $N$ parameter sets.
3. Run model for $N$ parameter sets and rank them.
4. Estimate model prediction bounds (e.g. $K=3$).
5. Estimate intersection $\lambda$ (e.g. for $K=3$, $\lambda=66\%$).
6. Repeat for $K=4, 5 \ldots N$ (find relationship between $\lambda$ and $K$).

$\lambda_{max}$ for $K=3$ is $66\%$.

Top 30 are the behavioural parameter sets.
Step 7

1. Define uncertainty intervals for each observed datapoint
2. Generate $N$ parameter sets
3. Run model for $N$ parameter sets and rank them
4. Estimate model prediction bounds (e.g. $K = 3$)
5. Estimate intersection $\lambda$ (e.g. for $K = 3$, $\lambda = 66\%$)
6. Repeat for $K = 4, 5 ... N$ (find relationship between $\lambda$ and $K$)
7. Use Step 6 to make a ‘less’ subjective cut-off and perform uncertainty assessment
Case-study

- MOPUS Rainfall-Runoff model

### Impervious component

![Diagram of Impervious component]

- $I(t)$
- $I_{imp}(t)$
- $I_{PSC}(t)$
- $Q_{imp}(t)$

#### 6 Parameters
1. IMP - Imperviousness
2. IT - Impervious store cap.
3. PSC - Pervious store capacity
4. $k$ - Routing coefficient
5. $m$ - Routing exponent
6. TOC

### Pervious component

![Diagram of Pervious component]

- $PervEvap(t)$
- $I(t)$
- $S_{imp}(t)$

\[ Q(t) = k \cdot \text{RoutingStore}^m \]

\[ Q_{outlet}(t) = Q(t - TOC) \]
Case-study

- Clayton catchment; 2 years of continuous flow and rainfall

<table>
<thead>
<tr>
<th>Land use</th>
<th>Light-industrial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>28 ha</td>
</tr>
<tr>
<td>Total imperviousness</td>
<td>80%</td>
</tr>
<tr>
<td>Catchment slope</td>
<td>1%</td>
</tr>
<tr>
<td>Rainfall gauge distance from outlet</td>
<td>300 m</td>
</tr>
<tr>
<td>Range of event rainfall totals</td>
<td>2.0 – 25.4 mm</td>
</tr>
</tbody>
</table>

**Number of rainfall events** 108
Case-study

1. Define uncertainty intervals for each observed datapoint
2. Generate $N$ parameter sets
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5. Estimate intersection $\lambda$ (e.g. for $K=3$, $\lambda=66\%$)
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7. Use Step 6 to make a ‘less’ subjective cut-off and perform uncertainty assessment
Results – cut off threshold

- Not possible to cover all observations - ?
- Max. intersection of 97% (top 500 parameter sets)
- Performance not linearly proportional to number of parameter sets
Results – Parameter distributions

- Flat distribution???
Results – Uncertainty bounds

- Good intersection between uncertainty bounds and measurement bounds

Now I get it!!!...97% intersection between model and measurement bounds
Results – Uncertainty bounds

- Low flows: model uncertainty lower than measurements’
- Above 750 l/s: model uncertainty explodes
- 99% observations below 750 l/s
Conclusions

• A new approach to conduct uncertainty analysis

• Subjectivity is reduced by proposing a tangible criterion

• Measurement uncertainty is taken into account

• Wider application of uncertainty analysis in the “real world” (hopefully)

We love the VMC!
Future work

• Validation

• More catchments

• Water quality model

• Input uncertainty (e.g. time displacement in rainfall)

• Exposure of the proposed method to ”real practitioners”