GLOBAL SENSITIVITY ANALYSIS FOR URBAN WATER QUALITY MODELLING: COMPARISON OF DIFFERENT METHODS

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Introduction

Only few studies in urban drainage deal with GSA
Previous GSA studies mainly focused on other research fields
Limited transferability of the knowledge derived from other fields
There is a need to experience with GSA methods in urban drainage to give answers to:

What is the best method? Is any??
What are the main features of each method?
How similar are the results?
To what extend do results differ changing method?
Aims of the study

Gain insights about the features of three GSA methods:

- SRC
- Morris screening
- E-Fast

Urban drainage water quality model
Case study
Comparison between the results of each method
Introduction

Terminology
Introduction: Problem statement

A complete, Clear, Generally Accepted Definition Is lacking!!

Suggest a common terminology on the basis of previous studies
Sensitivity methods: making a framework

Factors may include both model inputs and model parameters.
Methods
Methods

Standardized Regression Coefficients

Consists of running a Monte Carlo simulation and multivariate linear regression between the model outputs and inputs.

\[ SRC(x_i) = \beta_i = b_i \cdot \sigma_{x_i} / \sigma_y \]

\( \sigma_{x_i} \) and \( \sigma_y \) represent respectively the factor and the model output standard deviation and \( b_i \) the regression slope.

**SRCs are valid measures of sensitivity when \( R^2 > 0.7 \) (Saltelli et al., 2004)!!**

\[ \sum (\beta_i) = 1 \]

Linear model

The sign of \( \beta_i \) indicates its positive (sign +) or negative (sign -) effect.
**Methods**

**Morris Screening**

It is based on a one-at-a-time (OAT) perturbation of the model input factors.

Factor space is partitioned into \( p \) levels.

OAT is repeated \( r \) times, computing the Elementary Effects (EEs) for the model output considering a perturbation \( \Delta \):

\[
EES(x_i) = \frac{y(x_i, \ldots, x_{i-1}, x_i + \Delta, x_{i+1}, \ldots, x_k) - y(x_i, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_k)}{\Delta}
\]

Sensitivity is described by the absolute mean \((\mu^*)\) and standard deviation \((\sigma)\) of the cumulative distribution of the \( r \)\*EEs.
**Methods**

The **Extended-FAST** is based on the **variance decomposition theorem** which states that the **total variance** of the model output (Var(Y)) may be **decomposed** into conditional variances.

\[
S_i = \frac{\text{Var}_x(E_{x_i}(Y|x_i))}{\text{Var}(Y)}
\]

**First order index**

Show by which percentage the variance would decrease if the corresponding factor was known.

\[
S_{Ti} = 1 - \frac{\text{Var}_{x\neg i}(E_{x_i}(Y|x_{\neg i}))}{\text{Var}(Y)}
\]

**Total effect index**

Show by which amount the variance would be reduced if all factors except for the ith were known.
Urban drainage model & Case study
Methods: Urban water quality model (Mannina&Viviani 2010)

Quantity Module

- Hydrographs at the inlet and at the outlet of the sewer system

Quality Module

- Pollutographs at the inlet and at the outlet of the sewer system
Methods: Urban water quality model (Mannina & Viviani 2010)

The cascade of two linear reservoirs in series and a linear channel allow to split the hydraulic phenomena in the catchment from those in the sewer system.
Methods: Urban water quality model (Mannina&Viviani 2010)

Sewer System model:

Quality module

Rainfall runoff

Flow routing through the network

Build-up

Wash-off

Sediment deposition

Sediment erosion

Catchment

Sewer
Model application: the case study

**The experimental catchment**  Montelepre Catchment (IT)

- 36 storm events during one year 2008
- Drained area 40 ha, with an impervious percentage of 75%
- The drainage network ends in a polycentric section pipe 144 cm high and 180 cm wide
Methods: Application and criteria for comparison

- The model run: 1 year series - 36 events

- Seven model outputs:
  - Maximum sewer flow rate (\(Q_{\text{MAX}}\))
  - Total sewer flow volume (\(V_{\text{TOT}}\))
  - Maximum TSS sewer concentration (\(C_{\text{MAX,TSS}}\))
  - Maximum BOD concentration (\(C_{\text{MAX,BOD}}\))
  - TSS sewer load (\(L_{\text{TOT,TSS}}\))
  - Average TSS sewer concentration (\(C_{\text{AVERAGE,TSS}}\))
  - Average BOD sewer concentration (\(C_{\text{AVERAGE,BOD}}\))

- Seventeen model input factors

- Quantity and quality input factors changed simultaneously for each MC

- Uniform distribution – input factor
Methods: Application and criteria for comparison

• For the SRC and Morris screening: CFT = 0.1

• For E-FAST: CFT1 = 0.01 [correspondence between $\beta_i^2$ and $S_i$ (Saltelli et al., 2000)]

• For E-FAST: CFT2 = 0.1 for the value of the interaction $S_{T_i} - S_i$

• For each method a rank of importance according to factors prioritisation

Comparison for factors prioritisation:

• $\beta_i^2$ and $S_i$ SRC vs E-FAST;

• $\beta_i^2$ and $\mu^*$ SRC vs Morris Screening;

• $\mu^*$ and $S_i$ Morris Screening vs E-FAST;

Comparison for factors fixing (Morris screening vs E-FAST):

• $\mu^*$ versus $S_{T_i}$

• $\sigma$ versus $S_{T_i}$
Results: SRC $(Q_{\text{MAX}} - C_{\text{MAX,BOD}})$

Quantity

Quality
Results: Morris screening \( (Q_{\text{MAX}} - C_{\text{MAX,BOD}}) \)
Results: E-FAST \( (Q_{\text{MAX}} - C_{\text{MAX}, \text{BOD}}) \)
Results: comparison among the methods

Factors Prioritisation
- Important Factors
  - SRC
  - Morris screening
  - E-FAST

Factors Fixing
- Non-Influential
  - Morris screening
  - E-FAST
Conclusions

- A comparison between three GSA methods: important, non-influential and interacting model input factors of an urban stormwater model;

- SRC method is inside its range of applicability for the outputs $Q_{\text{MAX}}$, $V_{\text{TOT}}$ and $L_{\text{TOT,TSS}}$ and outside for $C_{\text{MAX,TSS}}$, $C_{\text{MAX,BOD}}$, $C_{\text{AVERAGE,TSS}}$ and $C_{\text{AVERAGE,BOD}}$.

- The quality sub-module is non-linear: attention to non-influential factors.

- E-FAST allowed the quantification of the interactions of model factors.

- In terms of factor fixing similar results were obtained between Morris screening and E-FAST methods for $Q_{\text{MAX}}$.

- For $C_{\text{MAX,BOD}}$ the Morris screening showed some differences (semi-quantitative method!).
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