Dynamic Time Warping improves sewer flow monitoring

\[ \theta_{DTW} = (10 - 8)\Delta t = 2\Delta t \]
What’s the matter?
Quality control of discharge measurements

Problem

- Flow meters show considerable errors under *normal* operating conditions in sewers.

- Discharge
  - Ultrasonic flow meters: 10%
  - Tracer dilution methods: 6% to 16%
  - Venturi: 12% to 20%

- Ultrasonic *velocity* sensors
  - Single-point: 14 - 18%
  - Multi-point: 4 - 5%

Hoppe (2009), Smits (2008)
Quality control of discharge measurements

- Manual calibration in the lab or field is expensive.

- Usually only point calibration once per year or 3 months.

- During dry weather conditions!
Quality control of discharge measurements
Create independent information on flow velocities

Idea
• Use “natural” tracers in wastewater to obtain independent information on average flow velocities
⇒ Time shift of characteristic patterns between 2 measuring locations A and B contains information on travel time $\theta$. 
Quality control of discharge measurements
Create independent information on flow velocities

Idea

- Use “natural” tracers in wastewater to obtain independent information on average flow velocities

⇒ Time shift of characteristic patterns between 2 measuring locations A and B contains information on travel time $\theta$.

⇒ Length of the sewer section is obtained from map or field measurements.

$$v(t) \approx \frac{L}{\theta}$$
Methods
Ideal plug-flow reactor

Concept

\[ \theta(t) = \frac{V}{Q(t)} \]

Equations:

Packet A: \( 2\Delta t \rightarrow \Delta t(Q_1 + Q_2) = V \)
Packet B: \( 2\Delta t \rightarrow \Delta t(Q_2 + Q_3) = V \)
Packet C: \( 3\Delta t \rightarrow \Delta t(Q_3 + Q_4 + Q_5) = V \)
Packet D: \( 4\Delta t \rightarrow \Delta t(Q_4 + Q_5 + Q_6 + Q_7) = V \)

...
Ideal plug-flow reactor

4 Equations:

\[
\begin{align*}
\Delta t (Q_1 + Q_2) &= V \\
\Delta t (Q_2 + Q_3) &= V \\
\Delta t (Q_3 + Q_4 + Q_5) &= V \\
\Delta t (Q_4 + Q_5 + Q_6 + Q_7) &= V
\end{align*}
\]

Matrix notation: \( AQ = b \) with

\[
A = \begin{pmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 1 \\
\end{pmatrix},
\]

\[Q = (Q_1, Q_2, \ldots, Q_7)^T\] and \( b = \frac{V}{\Delta t} (1, 1, 1, 1)^T\)

How do we get the residence times of the water packets?
Dynamic Time Warping

• “Warps” two sequences non-linearly in the time domain so that the dissimilarity is minimized

• Was originally developed for speech recognition
• Is a standard technique for non-linear pattern matching

• Has been used to successfully estimate flow distribution in hydraulic flow dividers at WWTPs.

Dürrenmatt (2011)
Dynamic Time Warping

\[ d(A, B) = \sum_{w_k} D_{n,m} \]

(Keogh und Pazzani, 2000)
Dynamic Time Warping

Conditions for the warping path:
- Starts and ends in opposite corners
- continuous
- Steps are restricted
- Pattern appears first in A, then in B.
Dynamic Time Warping

Added stochasticity to avoid local optima:
1. Add noise to observations
2. Iterate computation of warping paths
Dynamic Time Warping

1. Add noise to observations
2. Iterate computation of warping paths
3. Compute average warping

\[ \theta_{DTW} = \theta \]
\[ \theta_{DTW} = \theta ? \]

- The estimated travel time does not equal the true travel time in real systems (Dispersion, Reaction).

Illustration: Tracer experiment

Error: \[ E = \frac{V}{t_\mu} - \frac{V}{t_m} \neq 0 \text{ if } t_\mu \neq t_m \]
Numerical experiments
Numerical experiments

- Testing the method on virtual data to determine the field of application

- “Benchmark Simulation Environment”
  - Inflow generator (Discharge, Temperature)
  - Hydrodynamic heat transport model (Aquasim)
  - Sensor model (BSM 1, Class “A”)
Results (1)
Example

Time [s]

(a) $Q_{true}$ [m$^3$/s]
$V_{true}$ [m/s]

(b) $T_{in}$ [°C]
$T_{out}$ [°C]
$T_A$ [°C]
$T_B$ [°C]
Results (1)

Example

(a) 

(b) 

(c) Average Travel Time [s]

(d)

Legend:
- $Q_{true}$ [m$^3$/s]
- $v_{true}$ [m/s]
- $T_{in}$ [°C]
- $T_{out}$ [°C]
- $T_{A}$ [°C]
- $T_{B}$ [°C]
- $v_{est}$ [m/s]
- $v_{est,acc}$ [m/s]
- $v_{true}$ [m/s]
- Std. dev [s]
Results (1)

Accuracy of DTW velocity estimates

Confirms the theoretical considerations.
Results (2)
Dispersion and heat exchange

![Graph showing dispersion and heat exchange with lines for different values of k.]

- $k = 1.8 \times 10^{-3}$ s$^{-1}$
- $k = 8.8 \times 10^{-4}$ s$^{-1}$
- $k \leq 4.4 \times 10^{-4}$ s$^{-1}$
Results (3)

Sensor response time and error

![Graph showing sensor response time and error]
Results (4)

Sampling frequency

![Graph showing sampling frequency with different time intervals (Δt=15 s, Δt=30 s, Δt=60 s).]
Application
Real-world case study

• Testing the performance of 2 flow meters

• Measurement campaign: 2 weeks

• 2x Onset TMC6-HD thermistor w. HOBO logger
  • Accuracy: 0.25 °C
  • Resolution: 0.03 °C
  • $T_{10/90}$: 30s (90%)
Results (5)

Online analysis

a) Measured temperature

b) Online analysis (both flow meters)
Results (6)
Offline analysis

c) Offline Analysis: Flow meter 1

- Rel. error
- 99% cov. int.

Velocity [m/s]

Rel. error [-]

-2.0 -1.5 -1.0 -0.5 0.0 0.5 1.0 1.5 2.0

0.3 0.4 0.5 0.6 0.7 0.8

Point density
This looks nice. But...
Discussion

• **Pre-processing** is important! High-pass filtering is better than normalization of the Temperature signals.

• Results should be improved by **using other or multiple tracers** with near-conservative behaviour (e.g., Conductivity).

• Using a **physically-based model** for data analysis could also be promising.

*Dürrenmatt, D.J., D. Del Giudice, J. Rieckermann et al., Dynamic time warping improves sewer flow monitoring (submitted to Water Research)*
Figure B.6: Comparison of the estimated velocity $v_{est}$ with the true velocity $v_{true}$ for the XCORR (left) and DTW method (right). Accepted data points are indicated, as well as the weighted average with the 99% coverage intervals. For this figure, a total of $N$ values within the 10% percentile of the standard deviation of the paths were accepted.
Conclusions
Conclusions

• **Dynamic time warping (DTW)** can retrieve sewer flow velocities from online measurements of wastewater quality.

• **DTW extracts travel times** from the temporal shift between upstream and downstream patterns by computing a non-linear warping path which maximizes the similarity between both patterns.

• The method is very well suited for the conditions found in typical sewer systems. Errors are estimated to **less than 7.5%**.

• The **simple set-up** and low experimental costs for sensors make it a practicable approach to diagnose sewer flow monitoring devices.