

Urban drainage uncertainty analysis: should we break our back for normally distributed residuals?

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Model calibration, sensitivity & uncertainty analysis

- Bayes' theorem

$$p(\theta|d) = \frac{p(d|\theta)p(\theta)}{p(d)}$$

model parameters θ

measured calibration data d

$p(\theta/d)$ posterior distribution of θ for given d

$P(\theta)$ prior distribution of θ before constraining θ through calibration

$p(d|\theta)$ likelihood function (probability of observing d for given θ)

$p(d)$ distribution of observations

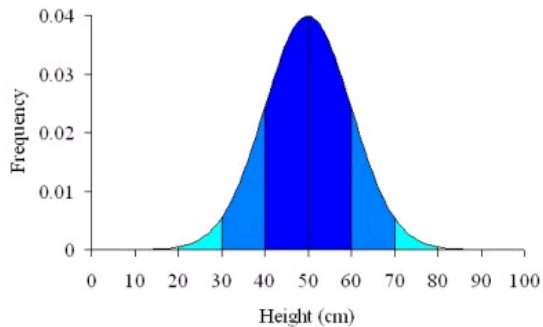
Model calibration, sensitivity & uncertainty analysis

Likelihood function

$$p(d|\theta) = \frac{\prod w_i \exp\left(-\frac{1}{2} \sum (d - M(\theta)_i w_i)^2\right)}{(2\pi)^{n/2}}$$

likelihood

residual



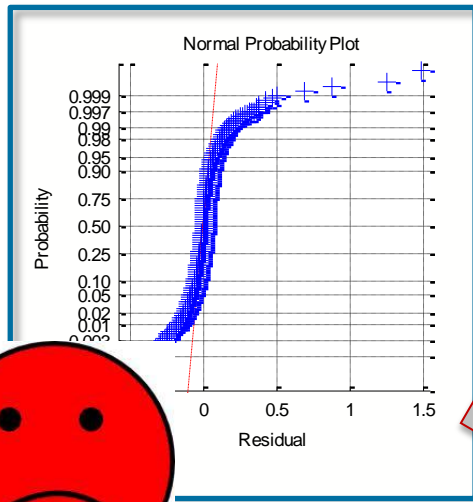
residuals

$[d - M(\theta)]_i$

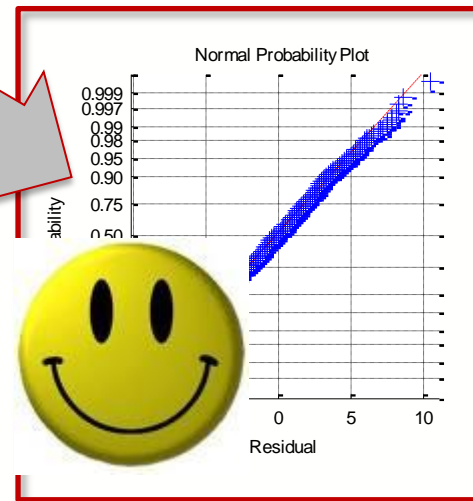
Residuals

Not checked
 $N(0, \sigma)$

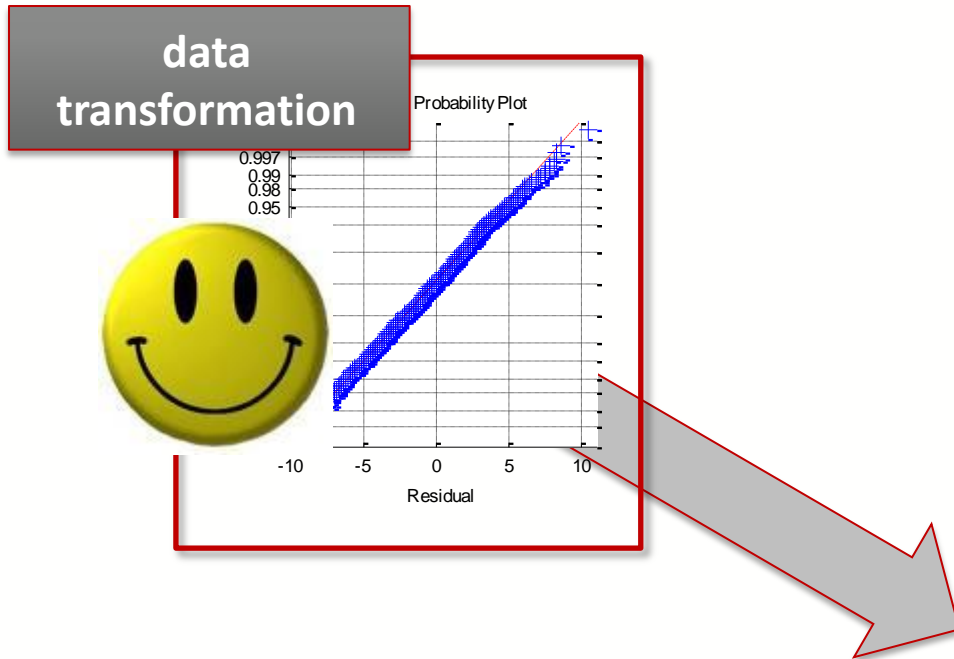
Residuals



data transformation



Residuals



change implied
information
(structure) of the
observations

model sensitivity

Aim

- impacts of verifying the normality assumption on the model sensitivity and associated parameter uncertainty



Models & Data

MUSIC

KAREN

- conceptual lumped models

- continuous simulation

- series of reservoirs

- simple linear reservoir

- flows impervious & pervious area

- flows impervious area

- 13 par

- 4 par

- 2 years rainfall & flow 6 min – urban catchment Melbourne (TIF 70%)

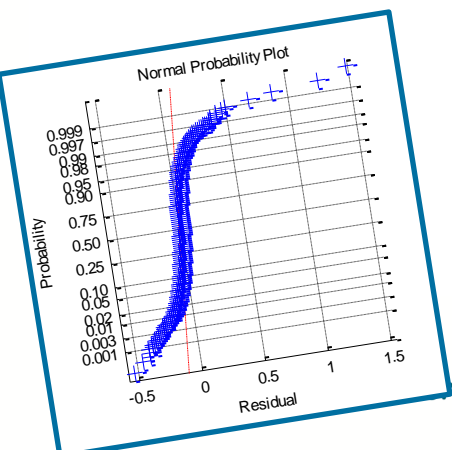
Scenarios

Scenarios

Scenarios

Not normal

Scenarios	
<i>Unverified</i>	normality of the residuals was checked but not verified

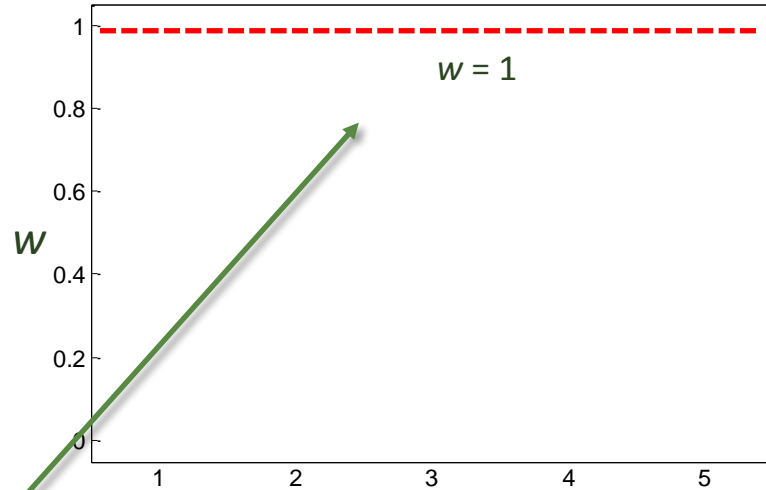


Positive Cities

Scenarios

Scenarios	
<i>Unverified</i>	r v t a
<i>Verified1</i>	r v t a
<i>Verified2</i>	r v t a

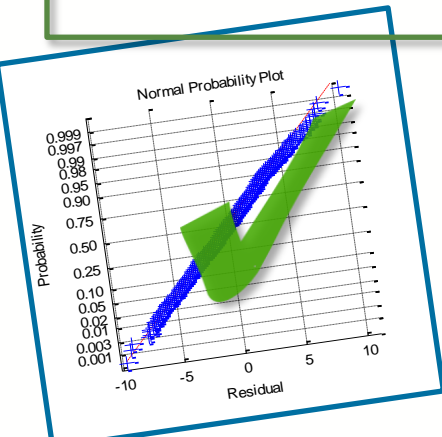
Not normal



$$Q^* = \frac{(Q + \lambda_2)^{\lambda_1} - 1}{\lambda_1}$$

Box-Cox transformation

normality assumption was verified, but no weights were applied to the data



Scenarios

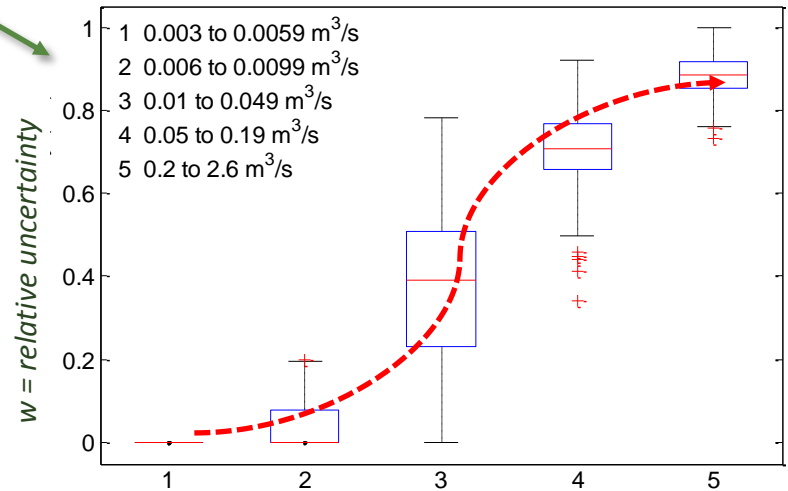
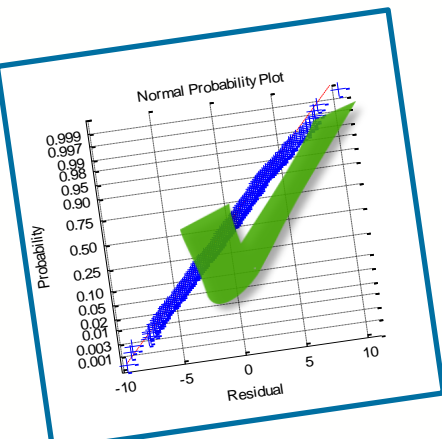
Scenarios	$w = 1$
<i>Unverified</i>	normality of the residuals was checked but not verified
<i>Verified1</i>	normality assumption was verified and a weighting strategy that gives more importance to high flows in the likelihood function was applied
<i>Verified2</i>	normality assumption was verified and no weighting strategy was applied

Not normal

Box-Cox transformation

$$Q^* = \frac{(Q + \lambda_2)^{\lambda_1} - 1}{\lambda_1}$$

**Normal & no weights
(w=1)**



Scenarios

Scenarios	
<i>Unverified</i> Not normal	normality of the residuals was checked but not verified
<i>Verified1</i> Normal & $w = \text{relat } u$	normality assumption was verified and a weighting strategy that gives more importance to high flows in the likelihood function was applied
<i>Verified2</i> Normal & no weights ($w=1$)	normality assumption was verified, but no weights were applied to the data

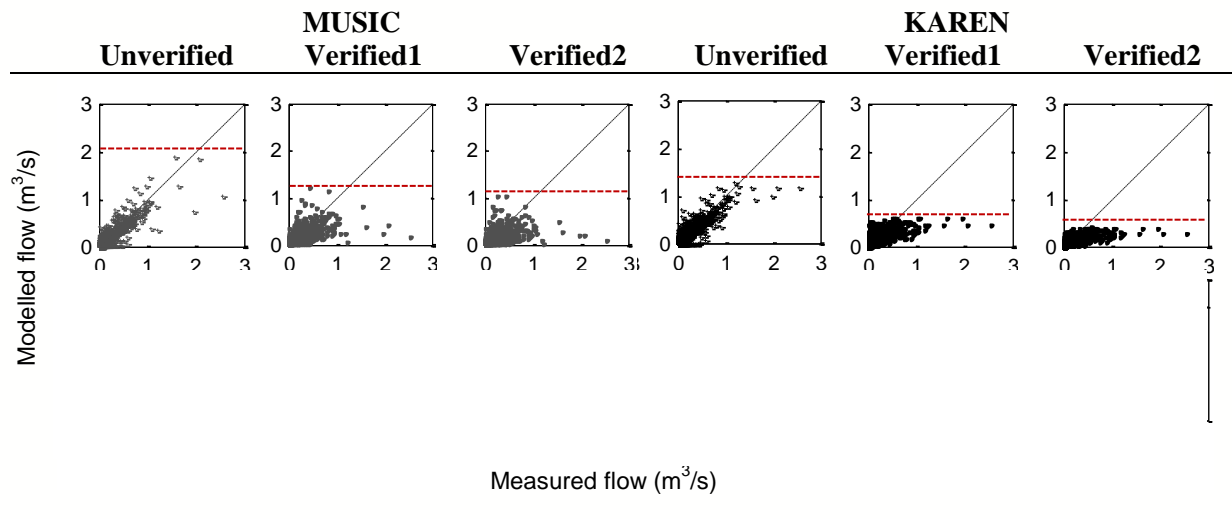
Results

Analysis of results

- Model efficiency
- Model sensitivity
- Prediction parameter uncertainty

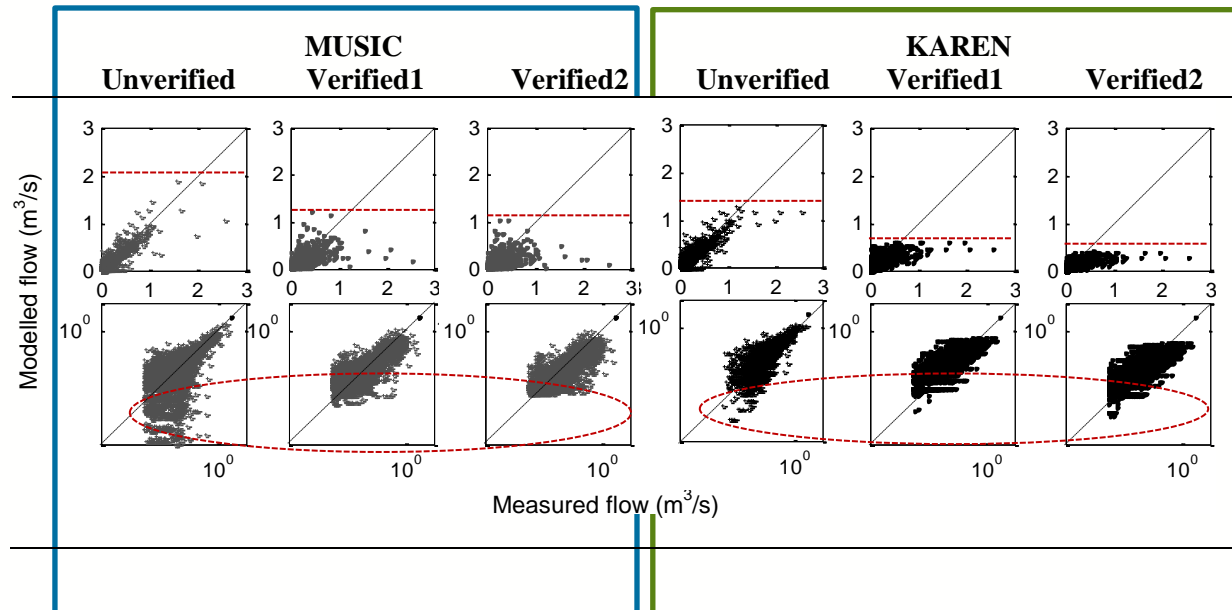
Results

Model efficiency (E)		
	MUSIC	KAREN
Unverified	0.81	0.80
Verified1	0.54	0.58
Verified2	0.48	0.46



Results

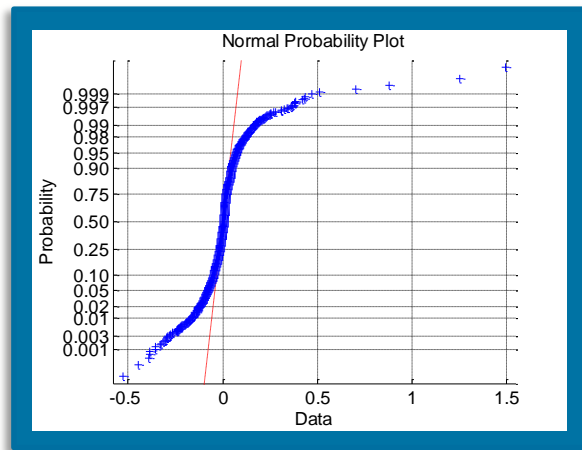
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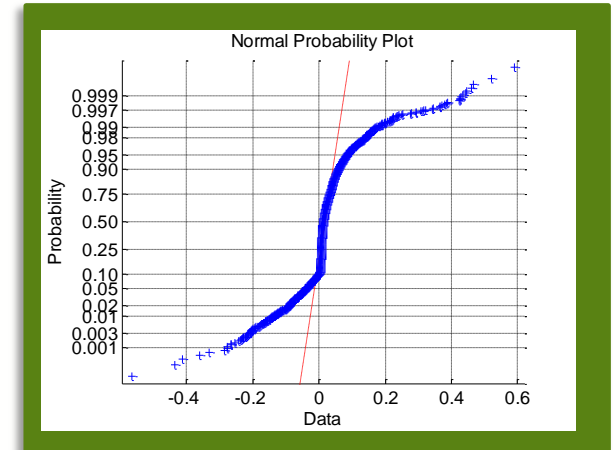
Results

Residuals – unverified

MUSIC



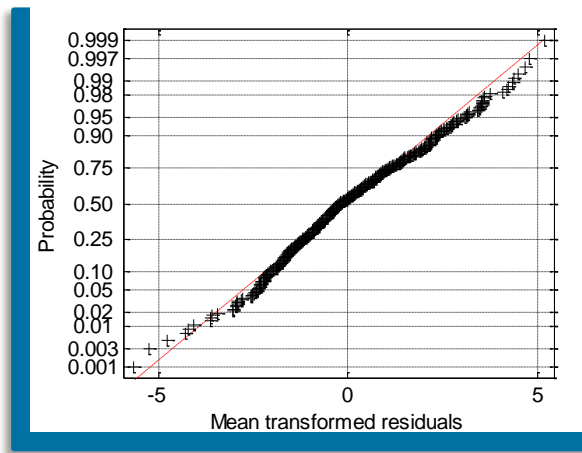
KAREN



Results

Residuals – *verified*

MUSIC

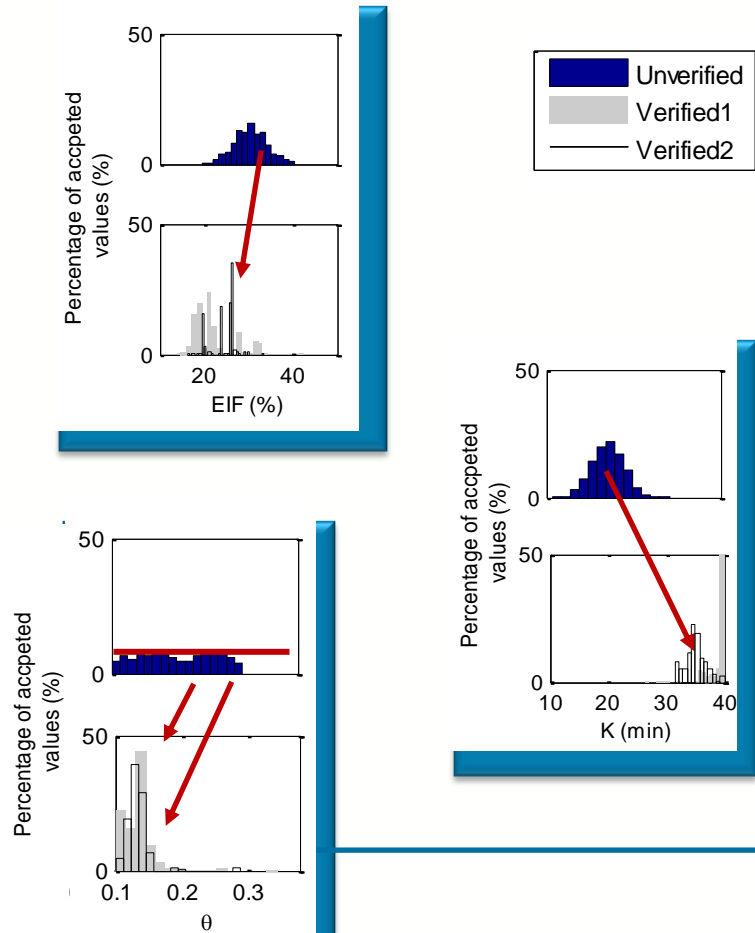


KAREN

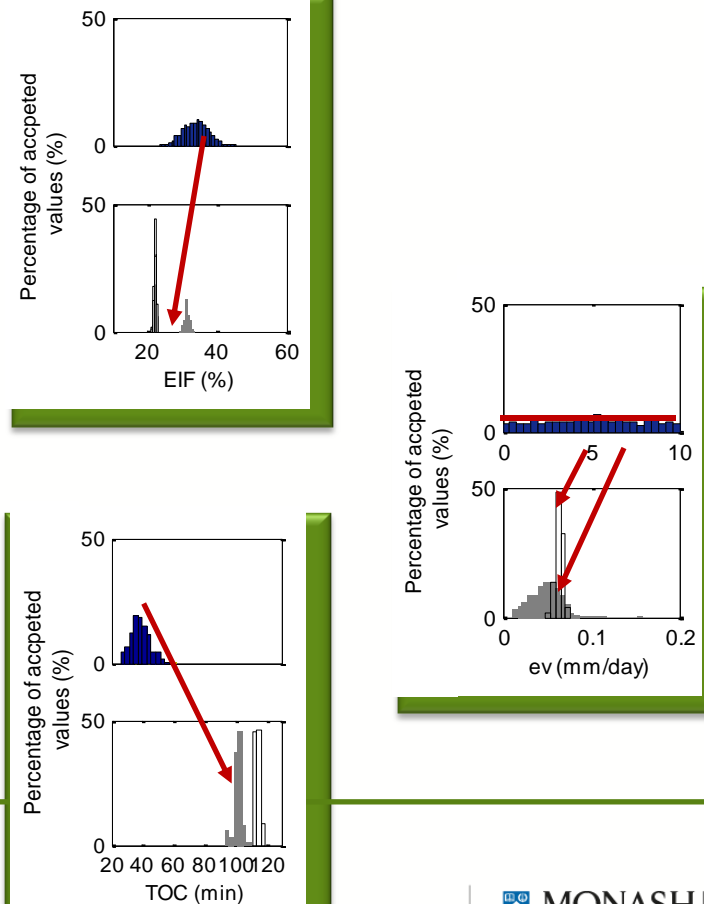


Results

MUSIC



KAREN

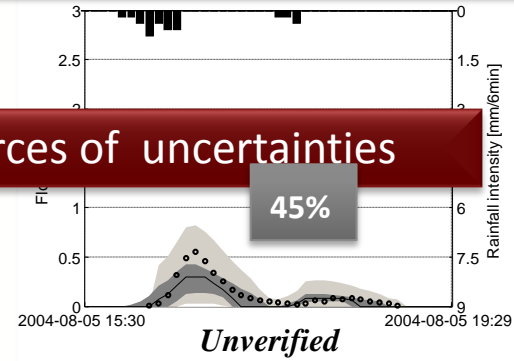
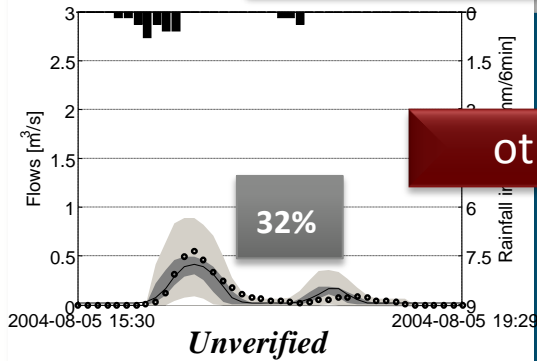


Results

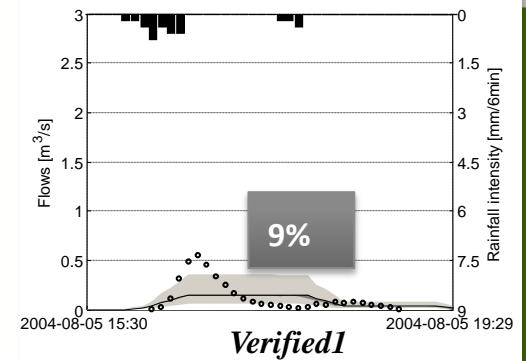
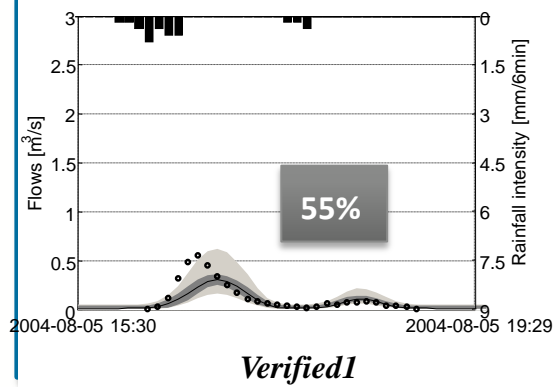
MUSIC

KAREN

n. observations within the parameter uncertainty bound (%)



other significant sources of uncertainties



unverified

verified1

verified2

black-box models

better calibrate models -
better predictions outside the calibration period

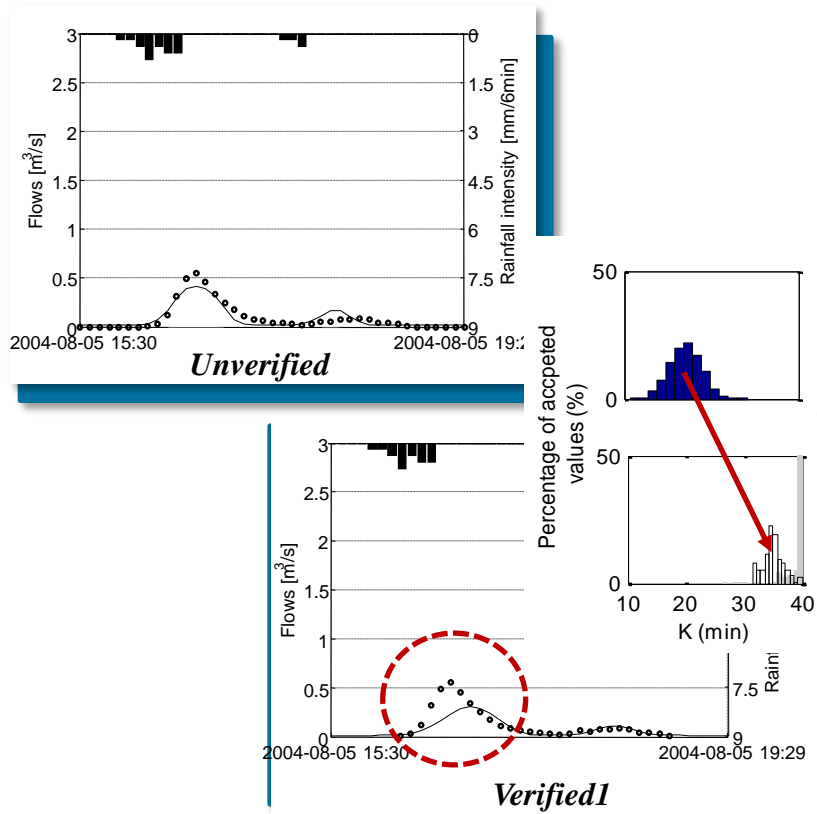
Info role param - model structure

Questions



Results

MUSIC



KAREN

