Urban drainage uncertainty analysis: should we break our back for normally distributed residuals?

Cintia. B. S. Dotto
Ana Deletic
David T. McCarthy
Model calibration, sensitivity & uncertainty analysis

- Bayes’ theorem

\[ p(\theta | d) = \frac{p(d | \theta) p(\theta)}{p(d)} \]

- Model parameters \( \theta \)
- Measured calibration data \( d \)
- \( p(\theta/d) \) posterior distribution of \( \theta \) for given \( d \)
- \( P(\theta) \) prior distribution of \( \theta \) before constraining \( \theta \) through calibration
- \( p(d | \theta) \) likelihood function (probability of observing \( d \) for given \( \theta \))
- \( p(d) \) distribution of observations
Model calibration, sensitivity & uncertainty analysis

Likelihood function

\[
p(d | \theta) = \frac{\prod w_i \exp \left( - \frac{1}{2} \sum (d - M(\theta)_i, w_i)^2 \right)}{(2\pi)^{n/2}}
\]

[Diagram showing likelihood and residuals]
Residuals

Not checked
N(0,\sigma)
Residuals

data transformation
Residuals

change implied information (structure) of the observations

model sensitivity

data transformation
Aim

- impacts of verifying the normality assumption on the model sensitivity and associated parameter uncertainty
Models & Data

MUSIC
- conceptual lumped models
- continuous simulation
- series of reservoirs
- flows impervious & pervious area
- 13 par

KAREN
- simple linear reservoir
- flows impervious area
- 4 par

- 2 years rainfall & flow 6 min – urban catchment Melbourne (TIF 70%)
### Scenarios

<table>
<thead>
<tr>
<th>Scenarios</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unverified normality of the residuals was checked but not verified</td>
</tr>
<tr>
<td>Verified normality assumption was verified and a weighting strategy that gives more importance to high flows in the likelihood function was applied</td>
</tr>
<tr>
<td>Verified normality assumption was verified, but no weights were applied to the data</td>
</tr>
</tbody>
</table>
## Scenarios

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Not normal</strong></td>
<td>Normality of the residuals was checked but not verified</td>
</tr>
<tr>
<td><strong>Unverified</strong></td>
<td>Normality assumption was verified and a weighting strategy was applied</td>
</tr>
<tr>
<td><strong>Verified</strong></td>
<td>Normality assumption was verified, but no weights were applied</td>
</tr>
</tbody>
</table>
### Scenarios

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unverified</strong></td>
<td>Normality of the residuals was checked but not verified.</td>
</tr>
<tr>
<td><strong>Verified1</strong></td>
<td>Normality assumption was verified and a weighting strategy was applied.</td>
</tr>
<tr>
<td><strong>Verified2</strong></td>
<td>Normality assumption was verified, but no weights were applied to the data.</td>
</tr>
</tbody>
</table>

#### Box-Cox transformation

$$Q^* = \frac{(Q + \lambda_2)^{\lambda_1} - 1}{\lambda_1}$$

#### Normal Probability Plot

**Residual Probability** vs. Normal Probability

- **w = 1**
- **w = 0.003 to 0.0059 m$^3$/s**
- **w = 0.006 to 0.0099 m$^3$/s**
- **w = 0.01 to 0.049 m$^3$/s**
- **w = 0.05 to 0.19 m$^3$/s**
- **w = 0.2 to 2.6 m$^3$/s**
Scenarios

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>$w = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unverified</strong></td>
<td>normality of the residuals was checked but not verified</td>
</tr>
<tr>
<td><strong>Verified1</strong></td>
<td>normality assumption was verified and a weighting strategy that gives more importance to high flows in the likelihood function was applied</td>
</tr>
<tr>
<td><strong>Verified2</strong></td>
<td>normality assumption was verified, but no weights were applied to the data</td>
</tr>
</tbody>
</table>

Box-Cox transformation

$$Q^* = \frac{(Q + \lambda_2)^{\lambda_1} - 1}{\lambda_1}$$

Normal & no weights ($w=1$)

- 1  0.003 to 0.0059 m$^3$/s
- 2  0.006 to 0.0099 m$^3$/s
- 3  0.01 to 0.049 m$^3$/s
- 4  0.05 to 0.19 m$^3$/s
- 5  0.2 to 2.6 m$^3$/s
### Scenarios

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Not normal</strong></td>
<td></td>
</tr>
<tr>
<td><em>Unverified</em></td>
<td>Normality of the residuals was checked but not verified</td>
</tr>
<tr>
<td><strong>Normal &amp; ( w = \text{relat} u )</strong></td>
<td></td>
</tr>
<tr>
<td><em>Verified1</em></td>
<td>Normality assumption was verified and a weighting strategy that gives more importance to high flows in the likelihood function was applied</td>
</tr>
<tr>
<td><strong>Normal &amp; no weights ( (w=1) )</strong></td>
<td></td>
</tr>
<tr>
<td><em>Verified2</em></td>
<td>Normality assumption was verified, but no weights were applied to the data</td>
</tr>
</tbody>
</table>
Results

Analysis of results

• Model efficiency
• Model sensitivity
• Prediction parameter uncertainty
### Results

#### Model efficiency (E)

<table>
<thead>
<tr>
<th></th>
<th>MUSIC</th>
<th>KAREN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unverified</td>
<td>0.81</td>
<td>0.80</td>
</tr>
<tr>
<td>Verified1</td>
<td>0.54</td>
<td>0.58</td>
</tr>
<tr>
<td>Verified2</td>
<td>0.48</td>
<td>0.46</td>
</tr>
</tbody>
</table>

---

#### Modelled flow (m$^3$/s)

<table>
<thead>
<tr>
<th></th>
<th>MUSIC</th>
<th>KAREN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unverified</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Verified1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Verified2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

#### Estimated flow (m$^3$/s)

<table>
<thead>
<tr>
<th></th>
<th>MUSIC</th>
<th>KAREN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unverified</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Verified1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Verified2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

#### Measured flow (m$^3$/s)

<table>
<thead>
<tr>
<th></th>
<th>MUSIC</th>
<th>KAREN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unverified</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Verified1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Verified2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Results

### Model efficiency (E)

<table>
<thead>
<tr>
<th>Model</th>
<th>MUSIC</th>
<th>KAREN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unverified</td>
<td>0.81</td>
<td>0.80</td>
</tr>
<tr>
<td>Verified1</td>
<td>0.54</td>
<td>0.58</td>
</tr>
<tr>
<td>Verified2</td>
<td>0.48</td>
<td>0.46</td>
</tr>
</tbody>
</table>
Results

Residuals – unverified

<table>
<thead>
<tr>
<th>MUSIC</th>
<th>KAREN</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Normal Probability Plot" /></td>
<td><img src="image2" alt="Normal Probability Plot" /></td>
</tr>
</tbody>
</table>

© CRC for Water Sensitive Cities 2012
Results

Residuals – verified

![Graphs showing mean transformed residuals against probability for MUSIC and KAREN models.](image)
Results

**MUSIC**

- EIF (%)
- Percentage of accepted values (%)
- SMax (mm)
- fc (mm)
- coeff

**KAREN**

- TOC (min)
- li (mm)
- ev (mm/day)
- gw (mm)
- dseep (x100%)
- K (min)
- 
Results

MUSIC

KAREN

n. observations within the parameter uncertainty bound (%)

other significant sources of uncertainties

Parameter uncertainty

CRC for Water Sensitive Cities

MONASH University
Results

Coverage from parameter uncertainties

<table>
<thead>
<tr>
<th>Parameters</th>
<th>MUSIC</th>
<th>KAREN</th>
</tr>
</thead>
<tbody>
<tr>
<td>EIF (%)</td>
<td>32</td>
<td>45</td>
</tr>
<tr>
<td>Unverified</td>
<td>55</td>
<td>9</td>
</tr>
<tr>
<td>Verified1</td>
<td>61</td>
<td>5</td>
</tr>
<tr>
<td>Verified2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Observations within the parameter uncertainty bound (%)

- **MUSIC**
  - Unverified: 32%
  - Verified1: 55%
  - Verified2: 61%

- **KAREN**
  - Unverified: 45%
  - Verified1: 9%
  - Verified2: 5%
Conclusions

unverified  verified1  verified2

black-box models

better calibrate models -
better predictions outside the calibration period

Info role param - model structure
Questions
Results

MUSIC

KAREN

CRC for Water Sensitive Cities

MONASH University

**Results**

Rainfall intensity
Total predictive uncertainty
Measured flow
Modelled flow

Rainfall intensity
Total predictive uncertainty
Measured flow
Modelled flow

**Metrics**

- Rainfall intensity
- Total predictive uncertainty
- Measured flow
- Modelled flow

**Verified**

**Unverified**

**Percentage of accepted values (%)**

**Flows [m^3/s]**

**Rainfall intensity [mm/6min]**

**K (min)**

**dseep (x100%)**

**g(w mm)**

**EIF (%)**

**SMax (mm)**

**fc (mm)**

**Coeff**